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Coordinating supply chain financing

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Coordinating Supply Chain Financing

by

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Abstract

Insufficient access to capital is one of the primary reasons for failure of existing and emerging businesses. This research investigates new coordination mechanisms for supply chains with one or more members under capital constraint. Supply chain profit and efficiency models are developed and optimized under different financing scenarios to explore the conditions that govern optimality. Coordination techniques are employed to make the financing agreement equitable for all supply chain members.

While supply chain finance has caught the attention of researchers recently, the little literature that does exist revolves around the conceptual framework and not the actual mathematical supply chain models analyzing the effects on profitability under stochastic conditions. This research advances the relatively new field of supply chain finance by providing conditions for optimality which will guarantee success. After first determining under which demand risk scenarios supply chain financing is beneficial, this research then explores supply chain financing optimal parameters and game theoretic coordination methods for a single period Newsvendor model. Finally, Mixed Integer Non-Linear Programming (MINLP) is utilized to coordinate supply chain financing in a multi-echelon, multi-period model. The results show that supply chain financing can be coordinated in a manner that yields higher profits and efficiencies for all supply chain members.

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Table of Contents

List of Tables

List of Figures

List of Abbreviations

SC	Supply Chain
SCF	Supply Chain Finance
SCC	Supply Chain Coordination
TNMF	Traditional Newsvendor Model with Financing
SS-TNMF	Supplier-led Stackelberg in Traditional Newsvendor Model with Financing
RS-TNMF	Retailer-led Stackelberg in Traditional Newsvendor Model with Financing
MWPC	Modified Wholesale Price Contract
SS-MWPC	Supplier-led Stackelberg With Modified Wholesale Price Contract
RS-MWPC	Retailer-led Stackelberg With Modified Wholesale Price Contract
TWPC	Traditional Wholesale Price Contract
SS-TWPC	Supplier-led Stackelberg With Traditional Wholesale Price Contract
RS-TWPC	Retailer-led Stackelberg With Traditional Wholesale Price Contract
RSLRD	Revenue Sharing Combing Loan Rate Discount
RSRWPC	Revenue Sharing Combing Revised Wholesale Price Contract

Chapter 1: Introduction

To improve global competitiveness, some enterprises choose to closely cooperate with other entities in the same supply chain and develop partner relationships with each other, being aware of the needs to optimize the whole chain rather than individual organizations. Influential companies such as IBM, Dell, Hewlett-Packard, Procter and Gamble have built long-term and collaborative relationships with their upstream suppliers to guarantee their market competitiveness [1] [2]. These supply chain partnerships typically have positive impacts; however during periods of budget constraints, tension occurs due to the differing, greedy viewpoints of buyers and suppliers with respect to finances. Buyers are trying to extend their payables to maintain their working capital level, which does harm to suppliers who cannot reclaim their debts in a certain period [3]. It brings forward the emerging field of supply chain finance (SCF) [4]. Supply chain finance is the merger of supply chain management and trade finance, and since the global financial crisis in 2008, it has started attracting the attention of practitioners and researchers. One result of today's globalization and offshore production is a complex and lengthened supply chain, where many enterprises have experienced significant reductions in capital availability. Today's global supply chain, where suppliers and buyers are spread globally, needs to explore alternative resources to external financing and utilize the existing capital within the supply chain to ease some of these burdens [5].

While supply chain finance is a relatively new concept, the definitions for supply chain finance vary greatly. Aberdeen Group defined SCF as "A combination of trade financing provided by a financial institution, a third-party vendor, or a corporation itself, and a technology platform that unities trading partners and financial institutions electronically and provides the financing triggers based on the occurrence of one or several supply chain events" [6]. SCF is developed to optimize working capital throughout the end-to-end supply chain for chain members [7]. Frohling [8] regarded SCF as a tool for small and medium-sized enterprises to solve financial problems. Later, some large multinationals that have strong credit ratings also realized the potential to increase the power of financial supply chain. Basu and Nair [9] believe the financial flows along the supply chains, which unlock trapped value, have become the essential part for relevant companies to continue their business operations. Supply chain financing influences supply chain profit and efficiency through the changing payment methods and periods. Therefore, well-managed supply chain finance is necessary for supply chain entities to run their business successfully.

For a capital-constrained supply chain entity, there are two common sources to financing capital: the third-party financial institution (i.e. bank) (external financing) and the upstream/downstream supply chain partner (internal financing). External financing requires the borrowers to have enough credit history and collateral [10], which is not often achievable if the borrower is of small size or a start-up. Also, the reliability of the third-party institution will affect the stability and safety of cash flow circulation in the supply chain. Internal financing generally takes two forms: the first involves the capital-constrained supply chain entity borrowing capital directly from its partner at an agreed upon interest rate with the assumption that the partner has enough available capital. The second is that the capital-constrained entity borrows capital from a financial institution and only pays the principal while its partner pays the periodic interest [11]. Which financing source to choose? The one that can bring higher profit and operation efficiency for the supply chain.

A supply chain consisting of independent enterprises should aim to increase the overall competitiveness of chain members. However, it is well documented that the supply chain does not realize its full profit potential due to greedy actions of each of the members [12] [13] [14]. SC members typically put their individual goal as a priority over the supply chain goal yielding sub-optimal supply chain performance. To overcome a low-efficiency supply chain, coordination mechanisms such as contracts, information sharing and information technology are needed to help coordinate supply chain actions. In today's economy, with an increasing number of start-up companies, financing management strategies must be identified to ensure their success [15]. According to CB Insights, 29% of startups fail because of bankruptcy [16]. Better coordination techniques are needed to guide capital financing. Therefore, the research topic of coordinating supply chain involved with financial issues is of vital importance and the research objective is to guide the supply chain consisting of one or more small businesses which confront financial crisis.

This research study consists of three parts. First, the benefit of internal financing is justified via mathematical models for a two-echelon supply chain consisting of a liquid supplier selling goods to a capital constrained retailer facing uncertain market demand. Three profit models are generated from three retailer options: order a sub-optimal quantity with limited capital with no borrowing, borrow capital from the third-party financial institution, or borrow capital from the upstream supplier. For each of these scenarios, profitability under various demand risk levels are analyzed as well as various market conditions governing the interest rates of the third-party financial institution, supplier, and the market investment. The second part extends the research to supply chain coordination with internal financing by developing profit and operation efficiency models for the supplier and retailer as functions of the financing terms and order quantities. Since the supplier and retailer are assumed to be risk-neutral, seeking to maximize their own profit and operation efficiency, a coordination mechanism is needed to eliminate the conflicts and achieve better supply chain performance. A new, modified wholesale price contract incorporating financing is employed to coordinate the supply chain actions, while non-cooperative game theory (Stackelberg Game) is applied to explore the optimal profit and efficiency equilibrium in different scenarios. In the last part, this study is extended to a more practical situation: coordinating a multi-echelon supply chain with multi-period financing. The studied supply chain consists of n entities $(n > 2)$, where one or more entities are short of working capital which will yield sub-optimal SC performance without proper financing and coordinating mechanism. Under a multi-period setting, the supply chain needs to determine the optimal contract periods to manage the chain business. The optimal contract period is determined through maximizing overall supply chain profit which will yield unsatisfactory profits and efficiencies among chain entities. A new coordination mechanism is proposed, combining revenue sharing implementing among the adjacent two entities that have no financial problems and the revised wholesale price contract implementing between financing provider and receiver. The coordination results are analyzed and compared in different scenarios.

With over 400 million individuals engaged in an entrepreneurial business around the world, finding best sources for financing is crucial [17]. Most of the current methods of supply chain financing are limited to trade financing which is not always available and do not typically optimize the supply chain. There are no existing quantitative models that determine optimal financing parameters, including interest rates charged inside the supply chain that also incorporate contracting terms. The value of the interest rate for supply chain internal financing affects the amount of capital borrowed as well as the product quantity to be available for market, which makes the optimal interest rate value worthy to explore.

The research contributions can be summarized into five aspects:

1. A comparison of supply chain financing options is explored for a two-echelon supply chain. New models for supply chain financing are developed which prove to outperform the no-financing model and can be widely adopted in relevant industries.
2. This research is the first to explore how the profits of two-echelon supply chain entities and the overall supply chain vary under stochastic demand risk. The results bring managerial insights that can guide the financing actions a supply chain should utilize under changing market demand.
3. A non-dominant Stackelberg game is developed for the Newsvendor model which results in optimal financing parameters. This coordination mechanism results in higher profits for both supply chain members.
4. A new coordination mechanism for a multi-echelon, multi-period financing scenario is developed incorporating revenue sharing and a revised wholesale price contract for financing one or more supply chain members. The resulting financing parameter optimization proves to improve the multi-echelon supply chain performance in a capital constrained situation. This methodology provides a new perspective to solve the supply chain financing related problems, adding profit and efficiency to the whole chain.

5. Little literature exists on multi-echelon supply chain financing issues. This research is the first to study how to coordinate multi-echelon supply chain with multi-period financing, which broadens horizon of supply chain application in both academia and industry.

Chapter 2: Background

SCF can take various forms such as trade credit, third party financing and direct supplier financing. Whatever the form, it is critical to many firms globally and has become more prevalent in the literature. This research is a form of trade credit, however, instead of extending payment terms, actual borrowing of capital by the retailer from either the supplier or a third party financier are discussed. Therefore, it is appropriate here to discuss both trade credit literature as well as supply chain financing literature.

2.1 Trade Credit

Trade credit is an agreement where the retailer is allowed to pay the supplier at a later date while discounts would be provided if the retailer would like to pay earlier and punishment would also be given if the payment is beyond the due date. Glassanos [18] put forward some advantages of trade credit, such as extending payment terms while enjoying a reliable supply base and easy access to lower cost of financing. Wang [19] explained that compared with third party finance, trade credit is more beneficial for supplier-led supply chains to achieve the most profits. However, these trade credit extensions may be correlated to the market power of the particular retailer, dependent upon their share of the supplier's business [20]. Thus a smaller retailer may not be able to obtain much trade credit and at the same time may face difficulties in obtaining affordable bank credit.

Fisman and Love [21] uncovered the significant role for trade credit as a source of firm financing and growth. They found that industries that are more dependent on trade credit financing grow more rapidly in the environment with poorly developed financial markets.

However, they did not compare the advantages and disadvantages of trade credit financing and third party financial intermediary financing. Jing and Seidmann [22] examined the relative merits of trade credit finance versus financing from a third party financial institution in a two-echelon supply chain consisting of a capital-constrained retailer and a manufacturer. They showed that when production costs are relatively low, trade credit finance is more effective than financing from the third party institution in mitigating double marginalization; otherwise, third party institution finance becomes more effective. Most retailers prefer to utilize trade credit rather than bank loans for short-term financing [23]. More theories and practices of trade credit are explained in [24] [25] [26].

As early as 1973, Haley and Higgins [27] explored inventory policies with trade credit financing, where purchased inventory can be considered to be financed in whole or in part with trade credit. They concluded that optimality requires order quantity and late payment time determined simultaneously in the context of lot-size model. Additional research examined the effects of trade credit on the optimal inventory policy, see [28] [29] [30] [31] [32]. Unlike the previous research articles which assume the retailer utilized trade credit from the supplier but did not pass along any savings to the customers, Huang [33] modeled the retailer's inventory system as a cost minimization problem to explore the optimal ordering policy under the assumption that the supplier would offer the trade credit to the retailer while the retailer also offers the trade credit policy to stimulate customer demand to develop the retailer's replenishment model. More theory and case studies of determining inventory policy of perishable items under trade credit financing has been discussed in [34] [35] [36] [37]. For more comprehensive review in the field of inventory policy with trade credit, refer to Soni et al. [38], Molamohamadi et al. [39].

2.2 Third Party Financing and Direct Supplier Financing

Third party institution financing is defined by a capital-constrained retailer who borrows certain amounts of capital from a third party financial intermediary at a given loan rate, which is usually not negotiable, while direct supplier financing is defined by the same capital-constrained retailer who borrows capital from the supplier, but at a negotiable rate. The research on third party institution financing and direct supplier financing is mainly studied by comparing it to trade credit financing [40] [41] [42] [43]. However, More and Basu [44] pointed out that both internal and external challenges exist to prevent implementing supply chain financing successfully. The internal challenges mainly focus on the structure and policy of the enterprise, and the employees while the external challenges refer to macroscopic political and social environment as well as the rapid development of science and technologies. However, lack of common vision among the supply chain partners is the most critical challenge confronting SCF. The possible solution is to integrate the SC members tightly to improve overall financial stability of the SC.

2.3 Game Theory in Supply Chain

There is vast literature on the use of game theory to analyze supply chains. Game theory, originally thought of as a mathematical approach, was developed in the 1940s by John von Neumann and Oskar Morgenstern [45]. In the following years, game theory was further developed by other researchers who extend game theory application into other disciplines such as social science. Myerson [46] defined game theory as “the study of mathematical models of conflict and cooperation between intelligent rational decision-makers”. Game theory has been applied in diverse areas such as economics, business, politics, etc. since

1940s and gradually moved to the field of operations research and management science in 1960s and 1970s. In the last two decades, researchers started to relate game theory with solving supply chain problems [47] [48] [49] [50]. To date, the game theory literature on supply chain analysis can roughly be divided into two types – cooperative (binding agreements allowed) and non-cooperative (no binding agreements allowed) game theory, which is more prevalent in the literature [51].

In non-cooperative game theory, heterogeneous supply chain actors only optimize their own objectives without caring for the effects of their decisions on the others, which needs a strategic equilibrium to determine rational outcome. Nash equilibrium [52] is a very popular equilibrium in which no player can benefit by unilaterally changing his/her strategy. A game may contain more than one Nash equilibrium. Unlike Nash equilibrium, Stackelberg strategy can avoid the equilibrium selection problem, and allow one actor to play as the leader who has one or more followers in the game. Recent studies include combining game theory with green supply chain to maximize economic and environmental benefits [53] [54] and the application in supply chain network optimization [55]. This research employs non-cooperative game theory to perform supply chain analysis, where Stackelberg strategy is discussed.

2.4 Supply Chain Coordination

Supply chain coordination is developed to unify the whole supply chain system and better deal with uncertainty in order to improve the overall performance of supply chain. Arshinder, Kanda, and Deshnukh [56] classify supply chain coordination mechanisms into four types – contracts, information technology, information sharing, and joint decision making. Coordination contracts are widely applied in practice. Govindan, Popiuc, and Diabat [57] gave a detailed summary about the supply chain coordination contracts (11 types), including wholesale price contract, two-part tariff, buyback, revenue sharing, quantity flexibility, and so forth. Cachon [12] described the function of a supply chain contract as to coordinate the supply chain to reach the set of supply chain optimal actions (Nash equilibrium), which implies the importance of game theory application in contract terms formulation. Cachon and Netessine [58] disseminated four types of game theory in supply chain coordination: cooperative static, cooperative dynamic, non-cooperative static, and non-cooperative dynamic games. They also explained the best response functions and the equilibrium for each game type. Nagarajan and Sosic [59] applied game theory to supply chain coordination and pointed out that different contractual agreements would be obtained along with different risk preferences (risk-neutral or risk-averse). In recent years, the application of green supply chain coordination [60], integrating carbon emission concerns into operational decision making [61], and incorporating emissions reduction incentives into supply chain coordination [62] have been studied. This research coordinates supply chain with modified wholesale price contracts, with the parameters as loan rate and wholesale price.

2.5 Supply Chain Coordination with Financing

This research topic belongs to the interfaces of operation and financing in supply chains where there are still problems remaining to be solved. Though a few works in the literature have addressed issues of studying a two-echelon SC over multiple periods, very little attention has been paid to coordinating mechanisms of a multi-echelon SC over multiple periods and none address this issue with regards to internal SC financing.

2.5.1 Two-Echelon, Single-Period Supply Chain Financing Coordination

The literature on coordinating the supply chain with financial constraints is recent. Kouvelis and Zhao [42] use Stackelberg strategy with supplier as the leader and retailer as the follower to handle the bankruptcy risk of both retailer and supplier. Their emphasis is to explore the retailer's better financing source (supplier or bank), analyze supply chain members' profits and the whole supply chain efficiency under optimal trade credit contract (supplier early payment discount scheme). Zhang and Tang [63] explored the wholesale price contract when a retailer's capital constraint is considered. Different scenarios of supply chain members' profits were analyzed when given different wholesale price and order quantity. Jaber and Osman [64] proposed a centralized model where players in a two-echelon supply chain coordinate their orders to minimize the cost of the supply chain, taking the permissible delay in payments as a decision variable. Lee and Rhee [65] employed a markdown allowance contract and derived the optimal markdown allowance and risk premium in the perspective of supplier, to fully coordinate supply chain. Yan [66] used a Stackelberg game to analyze the capital-constrained retailer's optimal ordering policy and the optimal wholesale price of the manufacturer in the perspective of credit line. Yan and Sun [43] published very similar research with Yan [66], taking into account supply chain coordination under financial constraints. Chen and Wang [67] presented a Newsvendor model where the supplier offered trade credit to the capital constrained retailer. The decision made by either the supplier or the retailer was related to the retailer's initial available working capital. However, trade credit was proven to create value for the whole supply chain coordination. For more research in the area of coordinating supply chain financing, refer to [68] [69] [70] [71] [72]. However, both of these mathematical models do not take into account the condition that capital-constrained retailer may borrow capital from either supplier or other financial institutions and there is no reference given to loan rate.

The work of Jing and Seidmann [22] is similar to this research. They utilize a two-echelon supply chain consisting of manufacturer and a capital constrained retailer and develop profit models under two different financing mechanisms (bank financing and trade credit financing). They conclude that which financing way is better subjects to the manufacturer's production cost. This work differs from Jing and Seidmann [22] in two aspects. First, the principal financed from either the third party financial institution or the supplier can be either optimal or suboptimal, with optimality based on profit maximization. Both of optimal and sub-optimal conditions of the financing amounts are incorporated into profit models for supply chain members. Second, rather than study profit from these models under fixed demand, the effect on profits of the supplier, retailer and supply chain with uncertain demand and varying demand risk levels are explored. The work of Yan and Sun [43] is also similar to this research. They start with the Newsvendor model and complete the supply chain financing model by adding a financial institution lending capital to constrained retailer based on the retailer's credit line. Stackelberg game with the manufacturer as the leader together with the wholesale price contract is explored to coordinate the supply chain. This work differs from Yan and Sun [43] in two aspects. First, rather than third party financing, internal financing where the retailer could borrow enough capital from liquid supplier is studied. Second, the two-level Stackelberg game as both supplier-led and retailer-led is analyzed, and optimal contracting parameters are developed. The proposed quantitative model will explore the conditions under which the operation efficiencies and profits of the three entities can increase, respectively.

2.5.2 Two-Echelon, Multi-Period Coordination Models

Linh and Hong [73] employ revenue sharing to coordinate a two-echelon supply chain model over two periods, in which the optimal value of revenue share ratio and wholesale price are explored in order to achieve channel coordination and a win-win outcome. Nishi and Yoshida [74] develop an algorithm to derive a Stackelberg equilibrium for optimizing multi-period two-echelon supply chain planning problem, which contributes to supply chain multi-period study. Peng, Zhou, and Wang [75] focus on coordinating multi-period coal-electricity supply chain under double price regulations using game models, through which coal fulfillment rate gets increased resulting in an increased order quantity and decreased electricity supply shortage. Kheljani, Ghodsypour, and Ghomi [76] combine a supplier selection model with a coordination model in a multi-period centralized supply chain, where the buyer selects and order from the right supplier while the supplier splits the ordered quantities into small lot size and make the delivery over multiple periods. A nonlinear programing model combined with revenue sharing contract is applied to maximize the overall supply chain profit. For more applications of coordinating multi-period supply chains in sourcing decisions, production networks, manufacturing decisions, please refer to [77], [78], [79].

2.5.3 Multi-Echelon, Single Period Coordination Models

The body of literature studying multi-echelon single-period supply chain coordination is rich. He and Zhao [80] employ traditional wholesale price contract combined with returns policy to coordination a multi-echelon supply chain under demand and supply uncertainty, which perfectly coordinated the supply chain. The proposed contract terms that lead to the win-win condition are explored through Nash bargaining analysis. A new revenue sharing contract between the most downstream entity and all upstream entities for multi-echelon supply chain [81] is studied and the advantages are well demonstrated by comparing with pairwise revenue sharing mechanism. Schoenmeyr [82] focuses on study inventory optimization through coordination mechanism and optimization algorithm design. He put forward the guaranteed service (GS) framework, in which base stock policies direct the operations of various supply chain stages, and then GS is proved to one another, which turn out to be an effective mechanism. Bergey [83] coordinated a three-party supply chain using procurement option contracts to yield significant benefits over more traditional fixed quantity contract and flexible buy-back contracts. More theories and practices of coordinating multi-echelon with single-period supply chain study are explained in [84], [85], [86], [87], [88].

2.5.4 Multi-Echelon, Multi-Period Coordination Models

The study on coordinating multi-period multi-echelon supply chain has started to attract attention recently. Sepehri [89] computes and analyzes the cost and inventory benefits along with multi-period and multi-echelon supply chain coordination, where bullwhip effect disappears and profit gets improved in cooperative supply chains. Chang [90] employs an integrated revenue sharing contract to coordinate a multi-period three-echelon supply chain, in which the retailer shares his revenue portion to manufacturer and distribution respectively meanwhile the distribution needs to lower the wholesale price. The optimality conditions of manufacturer, distributor, and retailer are explored as well as the equilibrium condition of the overall supply chain. Leong and Cheong [91] apply a combinatorial auction to coordinate multi-party multi-period supply chain, which smooths demands placed on supplier’s limited production capacities greatly and results in a supply chain of high efficiency. These current literatures focus on coordinating complex supply chains (either multi-period or multi-echelon) effectively but very few address supply chain financing related issues.

2.5.5 Supply Chain Financing Models

The existing literature on multi-period or multi-echelon supply chain financing is scarce. Zhou [92] modeled a stylized multi-period SC involving two parties where the retailer is capital constrained and manufacturer cooperates with bank to finance retailer. A Stackelberg game is used to explore the best financing scenario that can increase profits of both the retailer and manufacturer. Hasani and Khoshalhan [93] apply a credit option contract as an incentive scheme to encourage the manufacturer and buyer to participate in the coordination actions in a multi-period setting. The order quantity, credit time and backordered demand in each period function as decision variables, the optimal values of which are explored in different scenarios to examine the sensitivity of supply chain profit and contract efficiency. However, most of the current literatures in this domain focus only on finance in the perspective of management without introducing coordination mechanisms which is necessary.

The above literature review illustrates that coordination mechanisms between single-period/multi-echelon models, multi-period/Newsvendor type models do exist, however not within the realm of SC financing. The little literature that does exist in the area of SC financing mainly revolves around the interfaces of supply chain operation and finance. There is no literature found in the field of coordinating multi-echelon multi-period supply chain. Therefore, this research is one among the first to address issues in this domain. The methodology and managerial insights will possess referential value and contribute to the existing literature. Practically, the methodology proposed can enable a means of financing a resource-constrained entity within the supply chain such as a small/start-up business allowing them to compete in the marketplace, without undue risk to the financer.

Chapter 3: Methodology

Mathematical modeling is used to formulate the profit and operation efficiency models of supply chain entities under different financing and non-cooperative game scenarios. Then profit and efficiency are optimized to explore the conditions that govern optimality. The methodology and initial work are presented.

3.1 Generalized Two-Echelon SCF Model

This research studies a two-echelon supply chain with a capital-constrained retailer and its sole supplier, see Figure 3-1. The supplier is responsible for providing goods to the retailer, who purchases these goods from supplier at one price and sells them to final customers at a different price. The difference between the two prices is the profit per unit for retailer. In the traditional non-financing one-period model, the optimal order quantity for the retailer is determined by the Newsvendor model. However, in this research, it is assumed that the retailer is currently in a capital-constrained situation and cannot afford the optimal quantity of goods determined by maximizing the retailer's profit.

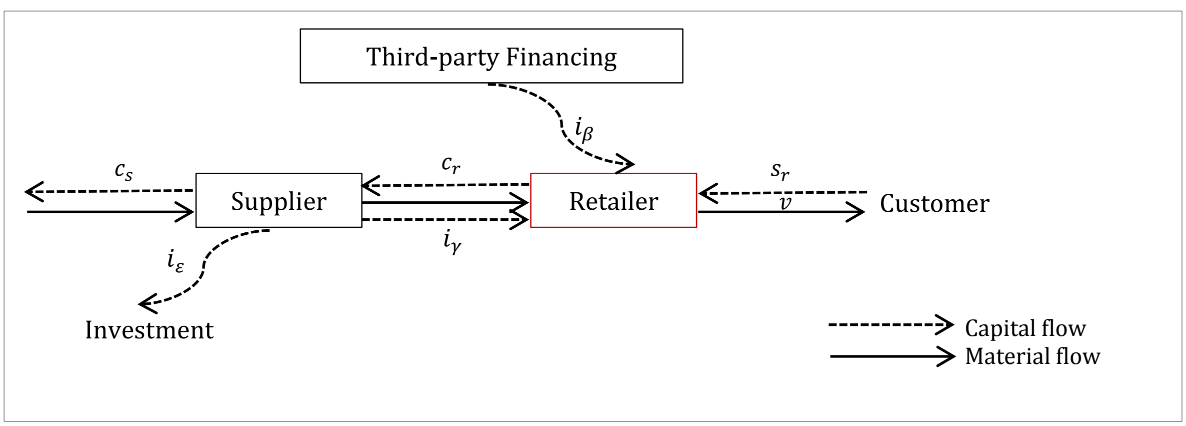

Figure 3-1: Two-Echelon Supply Chain Financing Model

Three cases will be discussed. In Case I, the retailer uses the limited capital on hand to purchase the sub-optimal quantity. This will yield in sub-optimal profits for the retailer and the supply chain. In Case II, the retailer borrows capital from a third-party financial institution at a fixed rate that is decided by the retailer's credit ratings, in order to purchase the optimal order quantity. In Case III, the retailer borrows capital from the liquid supplier at another rate which is changing and can be negotiated between retailer and supplier, and use it to again purchase the optimal order quantity from the same supplier.

Below are the model assumptions, which are consistent with the classic Newsvendor assumptions [94] [95]:

1. Customer demand is normally distributed with mean μ_1 and standard deviation σ_1. Demand risk is given by the coefficient of variance $\frac{\sigma_1}{\mu_1}$.

2. Supplier is unconstrained, and there is no delivery lead time and setup cost.

3. The interest rate charged by the third-party institution, i_β is constant, while the interest rate charged by the supplier, i_γ follows a uniform distribution U$(a, b]$, with $b < i_\beta$. Since the supplier stands to gain more profits if the retailer orders more, it is assumed that, the supplier will offer a rate which is lower than that of the third party institution, thus $b < i_\beta$. Both interest rates, once determined, are assumed to stay constant throughout the loan period.

4. Supplier has other market investing opportunities. The money that is lent to retailer can also instead be used for investing in the market. Supplier's market investment rate follows a normal distribution $N(\mu_2, \sigma_2)$ with the mean value $\mu_2 = i_\beta$ and the standard

deviation σ_2 changes with the demand risk level. Unsold units at the end of the period are sold at salvage value, v, while all stockouts result in lost sales at cost g_s and g_r for the supplier and retailer, respectively.

3.1.1 Definitions and Notation

The parameters and variables used in this model are shown in Table 3-1 below. Since this is a one-period model, simple interest is used to calculate the interest charged by the third-party financial institution and supplier. Also, given the retailer's financial state, the amount requested by the supplier may be larger than what their current credit status dictates, therefore, maximum and actual borrowing principals are given separately.

Table 3-1: Parameters and Variables: Two-Echelon SCF Model

s_r	Retailer's selling price per unit under normal condition ($/unit)
c_r	Retailer's unit cost ($/unit)
c_S	Supplier's unit cost ($/unit)
s_s	Supplier's selling price per unit ($/unit)
v_r	Retailer's salvage value ($/unit)
g_s	Goodwill (lost sale) cost per unit for supplier ($/unit)
g_r	Goodwill (lost sale) cost per unit for retailer ($/unit)
μ_1	Mean of customer demand
σ_1	Standard deviation of customer demand
D	Customer demand ~N(μ_1,σ_1)
M_∂	The retailer's limited amount of capital ($)
M_β	The amount of capital lent to retailer by the third-party financial institution ($)
M_γ	The amount of capital lent to retailer by supplier ($)
R_β	The maximum principal lent to retailer by the third-party financial institution ($)
R_γ	The maximum principal lent to retailer by supplier ($)
i_β	Interest rate per year charged by the third-party financial institution (%/year)
i_γ	Interest rate per year charged by supplier (%/year)
i_ε	Market investment rate of supplier per year (%/year) ~N(μ_2,σ_2)
μ_2	Mean of market investment rate
σ_2	Standard deviation of market investment rate
t	Time period (years)
π_s	Supplier Profit ($)
π_r	Retailer Profit ($)
π_{sc}	Supply Chain Profit ($)
Q_x	The order quantity in Case x ($x = I, II, III$)

3.1.2 Supply Chain Financing Options

The retailer is faced with three different financing options:

- Case I: retailer chooses not to borrow capital.
- Case II: retailer chooses to borrow capital from the third-party financial institution.
- Case III: retailer chooses to borrow capital from upstream supplier.

In each of these cases, the effects of profit on the supplier, retailer, and supply chain are investigated. The supply chain profit is calculated as the profit summation of the retailer and supplier. The profit models developed for different cases are demonstrated below.

3.1.2.1 Case I: No Financing

In Case I, retailer does not borrow capital from either the third-party financial institution or supplier, however, can only afford to buy limited amount, Q_I<Q*, the optimal order quantity given by the Newsvendor model. To maximize the retailer's profit in this case, the retailer should spend all his limited capital buying items from the supplier (see Lemma 1), and use ($Q_I = \frac{M_\partial}{c_r}$) to represent the order quantity without financing. The profit models for retailer, supplier, and supply chain are developed, respectively (Equations 3.1-3.3). (Note: subscript x-y indicates supply chain member x and case y).

$$\pi_{r-I} = s_r min(Q_I, D) + v_r max(Q_I - D, 0) - g_r \max(D - Q_I, 0) - M_\partial \tag{3.1}$$

$$\pi_{s-I} = (s_s - c_s)Q_I - g_s \max(D - Q_I, 0) \tag{3.2}$$

$$\pi_{sc-I} = \pi_{r-I} + \pi_{s-I} \tag{3.3}$$

Lemma 1. ***The capital constrained retailer's profit is monotonically increasing in order quantity without financing from either supplier or third party financial institution.***

According to Equation 3.1, the first derivative between retailer's profit and order quantity is calculated:

$$\frac{d(\pi_{r-I})}{d(Q_I)} = s_r + g_r - (s_r + g_r - v_r)F(Q_I), \textit{ where } F(Q_I) \textit{ is the cdf of } Q_I.$$

Since $s_r+g_r > (s_r+g_r - v_r)$ and $0 < F(Q_I) < 1$, *then* $s_r+g_r > (s_r+g_r - v_r)$ $F(Q_I)$. *Thus* $\frac{d(\pi_{r-I})}{d(Q_I)} = s_r+g_r - (s_r+g_r - v_r)F(Q_I) > 0$

Therefore, the retailer's profit is monotonically increasing in order quantity Q_I.

Proposition 2. ***The profits of the retailer, supplier, and supply chain are not sensitive to the changing demand risk levels in Case I.***

In order to investigate the effect on profits with increasing demand, two risk levels are used: Risk level A and Risk level B. Assume that $\mu_A = \mu_B$ and σ_A is slightly less than σ_B, so that the risk is slightly greater in risk level B and the effect of ΔD, $(D_A - D_B)$, is assumed to be small.

Demand risk level A: $\frac{\sigma_A}{\mu_A}$, the demand D_A , order quantity Q_A, the retailer's profit π_{r_A} , the supplier's profit π_{s_A} , the supply chain's profit π_{sc_A}. Demand risk level B: $\frac{\sigma_B}{\mu_B}$, the demand D_B , order quantity Q_B , the retailer's profit π_{r_B}, the supplier's profit π_{s_B}, the supply chain's profit π_{sc_B}.

Without financing, $Q_A = Q_B = \frac{M_\partial}{c_r}$, or simply, $Q = \frac{M_\partial}{c_r}$. The two scenarios for demand versus order quantity are investigated:

1) When $Q < D$,

$$\pi_{r\_A} = s_r Q - g_r(D_A - Q) - M_\partial$$

$$\pi_{r\_B} = s_r Q - g_r(D_B - Q) - M_\partial$$

$$\pi_{s\_A} = (s_s - c_s)Q - g_s(D_A - Q)$$

$$\pi_{s\_B} = (s_s - c_s)Q - g_s(D_B - Q)$$

Therefore,

$$\pi_{r\_A} - \pi_{r\_B} = g_r(D_B - D_A) = -g_r \Delta D$$

$$\pi_{s\_A} - \pi_{s\_B} = g_s(D_B - D_A) = -g_s \Delta D$$

$$\pi_{sc\_A} - \pi_{sc\_B} = (g_r + g_s)(D_B - D_A) = -\Delta D(g_r + g_s)$$

2) When $Q > D$,

$$\pi_{r\_A} = s_r D_A + v_r(Q - D_A) - M_\partial; \pi_{r\_B} = s_r D_B + v_r(Q - D_B) - M_\partial$$

$$\pi_{s\_A} = (s_s - c_s)Q;\ \pi_{s\_B} = (s_s - c_s)Q$$

Therefore,

$$\pi_{r\_A} - \pi_{r\_B} = (s_r - v_r)(D_A - D_B) = \Delta D(s_r - v_r)$$

$$\pi_{s\_A} - \pi_{s\_B} = 0$$

$$\pi_{sc\_A} - \pi_{sc\_B} = (s_r - v_r)(D_A - D_B) = \Delta D(s_r - v_r)$$

Given the nature of ΔD, it is concluded that under the scenario of no financing, the profits of the retailer, supplier, and supply chain are not sensitive to the changing demand risk levels.

3.1.2.2 Case II: Supply Chain External Financing

In Case II, it is assumed again that the retailer only has limited capital to purchase, $\frac{M_\partial}{c_r}$. However, the retailer requests to borrow, M_β from the third party financial institution at a loan rate i_β per period. The value of i_β , which is decided by retailer's credit history, stays constant over the financing period. The optimal amount of capital to borrow, M_β^* is decided by the optimal order quantity in Case II, Q_{II}^*, retailer's limited working capital, M_∂ and the cost per unit, c_r (see Equation 3.6). However, due to the credit history and collateral of the retailer, they may not be able to borrow that sum, therefore R_β is assumed to be the maximum amount of capital lent by the third financial institution, where $R_\beta \leq M_\beta^*$. After setting up the equations for Q_{II}^* and M_β^*, two scenarios are explored ($R_\beta = M_\beta^*$ and $R_\beta < M_\beta^*$). The retailer's profit model in Case II is given by:

$$\pi_{r-II} = s_r\, min(Q_{II}, D) + v_r max(Q_{II} - D, 0) - c_r Q_{II} - g_r max(D - Q_{II}, 0)$$
$$-(Q_{II} - \frac{M_\partial}{c_r})c_r i_\beta t \qquad (3.4)$$

To maximize the profit of retailer, the optimal quantity for the retailer to order is:

$$Q_{II}^* = F^{-1}\left(\frac{s_r - c_r + g_r - c_r i_\beta t}{s_r - v_r + g_r}\right) \qquad (3.5)$$

Therefore, the optimal amount of money to borrow from the third financial institution is:

$$M_\beta^* = (Q_{II}^* - \frac{M_\partial}{c_r})c_r \qquad (3.6)$$

Case II-a: $R_\beta = M_\beta^*$

In this scenario, the retailer should borrow M_{β}^{*} from the third party financial institution to bring the order quantity up to Q_{II}^{*} and achieve the maximum profit $\pi_{r-II}(Q_{II}^{*})$. The profits for the supplier and the supply chain are then calculated in Equations 3.7-3.9 respectively.

$$\pi_{r-II}(Q_{II}^{*}) = s_r\, min(Q_{II}^{*}, D) + v_r max(Q_{II}^{*} - D, 0) - c_r Q_{II}^{*} - g_r max(D - Q_{II}^{*}, 0) - (Q_{II}^{*} - \frac{M_{\partial}}{c_r}) c_r i_{\beta} t \quad (3.7)$$

$$\pi_{s-II}(Q_{II}^{*}) = (s_s - c_s) Q_{II}^{*} - g_s\, max(\mathrm{D} - Q_{II}^{*}, 0) \quad (3.8)$$

$$\pi_{sc-II} = \pi_{r-II}(Q_{II}^{*}) + \pi_{s-II}(Q_{II}^{*}) \quad (3.9)$$

Case II-b:$R_{\beta} < M_{\beta}^{*}$

In this scenario, the retailer cannot borrow the optimal amount M_{β}^{*}, but R_{β} at most. To maximize the profit with the borrowing capital no more than R_{β}, the retailer should borrow the maximum amount R_{β} to bring the order quantity to the relative optimal amount, Q_{II}'(see Lemma 3). The ordering quantity and profits for retailer, supplier and supply chain are shown in Equations 3.10-3.13.

$$Q_{II}' = \frac{R_{\beta} + M_{\partial}}{c_r} \quad (3.10)$$

$$\pi_{r-II}\ (Q_{II}') = s_r\, min(Q_{II}', D) + v_r max(Q_{II}' - D, 0) - c_r Q_{II}' - g_r max(D - Q_{II}', 0) - (Q_{II}' - \frac{M_{\partial}}{c_r}) c_r i_{\beta} t \quad (3.11)$$

$$\pi_{s-II}(Q_{II}') = (s_s - c_s) Q_{II}' - g_s\, max(D - Q_{II}', 0) \quad (3.12)$$

$$\pi_{sc-II} = \pi_{r-II}(Q_{II}') + \pi_{s-II}(Q_{II}') \quad (3.13)$$

Lemma 3. ***Salvage value can affect the existence of the retailer's optimal order quantity with financing from third financial institution, Q_{II}^*. If the salvage value is less than the interest-increased order cost (c_r(1+$i_\beta t$)), Q_{II}^* exists and is unique, otherwise the retailer's profit is monotonically increasing in the order quantity.***

Step 1. Based on Equation 3.4, the first derivative of retailer profit to order quantity is calculated:

$$\frac{d(\pi_{r-II})}{d(Q_{II})} = s_r - c_r + g_r - c_r i_\beta t - (s_r - v_r + g_r)F(Q_{II}), where\ F(Q_{II})\ is\ the\ cdf\ of\ Q_{II}.$$

When $v_r < c_r(1+i_\beta t)$, $s_r - v_r + g_r > s_r - c_r + g_r - c_r i_\beta t$.

Then, $0 < \frac{s_r - c_r + g_r - c_r i_\beta t}{s_r - v_r + g_r} < 1$

Therefore, Q_{II}^* exists and is calculated by making $\frac{d(\pi_{r-II})}{d(Q_{II})}$ equal 0:

Step 2. $$\frac{d^2_{\pi_{r-II}}}{d{Q_{II}}^2} = -(s_r - v_r + g_r)f(Q_{II}) < 0$$

The second derivative of π_{r-II} is negative indicating the concavity of the function. When $Q_{II} < Q_{II}^*$, π_{r-II} is monotonically increasing in Q_{II} while decreasing when $Q_{II} \geq Q_{II}^*$. Therefore, the optimal Q_{II}^* has a unique value.

Step 3. However, when $v_r \geq c_r$ (1+$i_\beta t$),

$\frac{d(\pi_{r-II})}{d(Q_{II})} = s_r - c_r + g_r - c_r i_\beta t - (s_r - v_r + g_r)F(Q_{II}) > 0$, indicating π_{r-II} is monotonically increasing in Q_{II}.

Proposition 4. ***When the retailer is able to borrow the optimal amount of principal from the third party financial institution, the profits of supplier increase with the increasing demand risk levels while the profit of retailer and supply chain have no identifiable trend to follow; when a sub-optimal principal is obtained, the profits of the retailer and supplier are not sensitive to the changing demand risk levels while supply chain profit has a relatively obvious but not identifiable trend.***

In order to investigate the effect on profits with increasing demand, two risk levels are used: Risk level A and Risk level B. Assume that $\mu_A = \mu_B$ and σ_A is slightly lower than σ_B, so that the risk is slightly greater in risk level B and the effect of ΔD, $(D_A - D_B)$ is assumed to be small.

Demand risk level A: $\frac{\sigma_A}{\mu_A}$, the demand D_A, optimal order quantity ${Q_A}^*$, the retailer′s profit π_{r_A}, the supplier′s profit π_{s_A}, the supply chain's profit π_{sc_A}. Demand risk level B: $\frac{\sigma_B}{\mu_B}$, the demand D_B, optimal order quantity ${Q_B}^*$, the retailer′s profit π_{r_B}, the supplier′s profit π_{s_B}, the supply chain's profit π_{sc_B}. Assuming $\frac{\sigma_A}{\mu_A} < \frac{\sigma_B}{\mu_B}$, then ${Q_A}^* < {Q_B}^*$ when $R_\beta = M_\beta^*$; while ${Q_A}^* = {Q_B}^* = \frac{R_\beta + M_\partial}{c_r}$ when $R_\beta < M_\beta^*$.

1) When the financing amount from the third party institution is optimal ($R_\beta = M_\beta^*$), the retailer will have enough capital to order from supplier, $Q \geq D$.

$$\pi_{r\_A} = s_r D_A - v_r(D_A - {Q_A}^*) - c_r {Q_A}^* - {Q_A}^* c_r i_\beta t + M_\partial i_\beta t$$

$$\pi_{r\_B} = s_r D_B - v_r(D_B - {Q_B}^*) - c_r {Q_B}^* - {Q_B}^* c_r i_\beta t + M_\partial i_\beta t$$

$$\pi_{s\_A} = (s_s - c_s){Q_A}^*$$

$$\pi_{s\_B} = (s_s - c_s){Q_B}^*$$

$\pi_{s\_A} - \pi_{s\_B} = (s_s - c_s)({Q_A}^* - {Q_B}^*) < 0$, indicating supplier profit increases with risk level.

$\pi_{r\_A} - \pi_{r\_B} = \left(v_r - c_r - c_r i_\beta t\right)({Q_A}^* - {Q_B}^*) + (s_r - v_r)\Delta D$, indicating there is no trend to be identified for retailer profit.

$\pi_{sc\_A} - \pi_{sc\_B} = \left(v_r - c_r - c_r i_\beta t + s_s - c_s\right)({Q_A}^* - {Q_B}^*) + (s_r - v_r)\Delta D$, indicating there is no trend to be identified for supply chain profit.

2) When the financing amount from third party institution is sub-optimal ($R_\beta < M_\beta^*$), retailer will not have enough capital to order from supplier, $Q < D$.

$$\pi_{r\_A} - \pi_{r\_B} = \left(s_r - c_r + g_r - c_r i_\beta t\right)({Q_A}^* - {Q_B}^*) - g_r\Delta D = -g_r\Delta D$$

$$\pi_{s\_A} - \pi_{s\_B} = (s_s - c_s + g_s)({Q_A}^* - {Q_B}^*) - g_s\Delta D = -g_s\Delta D$$

$$\pi_{sc\_A} - \pi_{sc_B} = -(g_r + g_s)\Delta D$$

Given the nature of ΔD, it is concluded that under this scenario of sub-optimal financing from third institution, the profits of the retailer and supplier are not sensitive to the changing demand risk levels while supply chain profit has a relatively obvious but not identifiable trend.

3.1.2.3 Case III: Supply Chain Internal Financing

In Case III, the retailer is borrowing capital from the supplier and wishes to borrow enough M_γ to bring the order quantity up to the optimal Q_{III}^*. Then the optimal amount of capital to borrow, M_γ^* is decided by Q_{III}^*, M_∂ and c_r (see Equation 3.16). Here R_γ is assumed to be the maximum amount lent by the supplier at interest rate i_γ for period *t*, where $R_\gamma \leq M_\gamma^*$. Similar to Case II, two different scenarios need to be discussed to finalize the order quantity, profit of the supply chain and chain members. The retailer's profit model in case III is given by:

$$\pi_{r-III} = s_r\, min(Q_{III}, D) + v_r max(Q_{III} - D, 0) - c_r Q_{III} - g_r max(D - Q_{III}, 0) - (Q_{III} - \frac{M_\partial}{c_r}) c_r i_\gamma t \quad (3.14)$$

To maximize the retailer profit, the optimal quantity for the retailer to order is:

$$Q_{III}^* = F^{-1}(\frac{s_r - c_r + g_r - c_r i_\gamma t}{s_r - v_r + g_r}) \quad (3.15)$$

Therefore, the optimal amount of capital to borrow from the supplier is:

$$M_\gamma^* = (Q_{III}^* - \frac{M_\partial}{c_r}) c_r \quad (3.16)$$

Case III-a: $R_\gamma = M_\gamma^*$

In this situation, the retailer should borrow M_γ^* from the supplier to bring the order quantity up to Q_{III}^* and achieve the maximum profit $\pi_{r-III}(Q_{III}^*)$. The profits for the retailer, supplier and the supply chain are then calculated, respectively in Equations 3.17-3.19.

$$\pi_{r-III}(Q_{III}^*) = s_r\, min(Q_{III}^*, D) + v_r max(Q_{III}^* - D, 0) - c_r Q_{III}^* - g_r max(D - Q_{III}^*, 0) - (Q_{III}^* - \frac{M_\partial}{c_r}) c_r i_\gamma t \quad (3.17)$$

$$\pi_{s-III}(Q_{III}^*) = (s_s - c_s) Q_{III}^* - g_s\, max(D - Q_{III}^*, 0) - (i_\varepsilon - i_\gamma)\, (Q_{III}^* - \frac{M_\partial}{c_r}) c_r \quad (3.18)$$

$$\pi_{sc-III} = \pi_{r-III}(Q_{III}^*) + \pi_{s-III}(Q_{III}^*) \quad (3.19)$$

Case III-b: $R_\gamma < M_\gamma^*$,

Here, the retailer cannot borrow the optimal amount M_{γ}^{*}, but R_{γ} at most. To maximize the profit with the borrowing capital no more than R_{γ}, the retailer should borrow the maximum amount R_{γ} to bring the order quantity to the sub-optimal amount, Q'_{III}. Order quantity, and profits for the three parties are shown in Equations 3.20-3.23.

$$Q'_{III} = \frac{R_{\gamma}+M_{\partial}}{c_r} \tag{3.20}$$

$$\pi_{r-III}(Q'_{III}) = \; s_r\, min(Q'_{III}, D) + v_r max(Q'_{III} - D, 0) - c_r Q'_{III} - g_r max(D - Q'_{III}, 0) - (Q'_{III} - \frac{M_{\partial}}{c_r}) c_r i_{\gamma} t \tag{3.21}$$

$$\pi_{s-III}(Q'_{III}) = (s_s - c_s)Q'_{III} - g_s\, max(D - Q'_{III}, 0) - (i_{\varepsilon} - i_{\gamma})\, (Q'_{III} - \frac{M_{\partial}}{c_r}) c_r \tag{3.22}$$

$$\pi_{sc-III} = \; \pi_{r-III}(Q'_{III}) + \pi_{s-III}(Q'_{III}) \tag{3.23}$$

Lemma 5. ***Salvage value will affect the existence of the optimal order quantity with financing from the supplier, Q_{III}^{*}. If salvage value is less than the interest-increased order cost c_r (1+ $i_{\gamma}t$), Q_{III}^{*} exists and is unique, otherwise, retailer profit increases with order quantity.***

See the proof for Lemma 5 in Appendix 1.

Proposition 6. ***When the retailer is able to borrow the optimal amount of principal from the supplier, the profit of supplier increases with the increasing demand risk levels while the profits of retailer and supply chain have no identifiable trend; when a sub-optimal principal is obtained, the profits of the retailer and supplier are not sensitive to the changing demand risk levels while supply chain profit has a relatively obvious but not identifiable trend.***

See the proof for Proposition 6 in Appendix 2.

3.1.3 Conclusions

The generalized models for the three scenarios a budget constrained retailer faces in a Newsvendor-like model (no financing, external third party financing or internal supply chain financing) under demand risks are developed. The consequences of each are explored and some important conclusions are drawn. In particular, it is important to note that profits of the retailer, supplier and supply chain are all insensitive to demand risk under the no financing scenario. With third party financing and supplier financing, when the retailer is able to borrow an amount which will enable the optimal order quantity to be purchased, only the supplier's profit increases with demand risk and the retailer and supply chain profits have no identifiable trend with the demand, however, when a suboptimal amount must be purchased, profits of both retailer and supplier are independent of risk. Finally under equal circumstances, internal supply chain financing will yield greater profits for the supply chain than third party financing. In Section 4.1, a numerical simulation is performed to demonstrate these results.

3.2 Game Theoretic Approach to Two-Echelon SCF

A two-echelon supply chain model is employed with a liquid supplier selling products to Newsvendor-like retailer facing uncertain demand and financial constraint conditions. The uncertainty can cause overstock or understock costs (backorders are not considered in this one-period model). Due to budget constraints, the retailer cannot afford the optimal order quantity derived from the market demand, so it opts to borrow capital from the upstream player at a certain loan rate. Note that this can also be construed as a form of trade finance whereby capital isn't actually passed from supplier to retailer but payment is delayed by the loan period, with the retailer paying interest at the same loan rate. However, both the retailer and supplier have the objective of maximizing their profits and efficiencies.

3.2.1 Notation and Assumption

The customer demand D is assumed to be independent of the fixed market selling price p_R, and follows a normal distribution with known parameter values- mean (μ) and standard deviation (σ). The retailer needs to pay c_R per unit that equals the wholesale price p_S. The retailer has limited working capital, R_R, and can only afford α, $(0 < \alpha < 1)$, portion of the optimal order quantity $Q*$. With internal financing, the retailer would borrow enough capital from the supplier to purchase (1-α) $Q*$. The understock and overstock costs for the retailer is g_R and v_R per unit, respectively. Understock cost g_R is assumed to be greater than the unit profit loss $(p_R - c_R)$, and $g_R = p_R - p_S + c_1$ where c_1 is the fixed cost. Similarly, the overstock cost v_R is assumed to be less than the unit purchase cost c_R, and $v_R = p_S + c_2$. The understock cost for the supplier is g_S per unit. Similar to g_R , g_S is assumed to be greater than the unit profit loss $(p_S - c_S)$ by a constant c_3. It is assumed that, the supplier's actual production quantity exclusively depends on retailer's order quantity. There is no overage cost for supplier since the retailer would take the risk. Please see notations listed in Table 3-2.

Other model assumptions are as follows:

1. The demand and cost information is shared completely between supply chain entities.
2. Both the supplier and retailer are risk-neutral.
3. The retailer will not default on the loan obligation (similar to the model assumption in [42]).

Table 3-2: Notations: Game Theoretic Model for Two-Echelon SCF

p_R	The retailer's unit selling price
p_S	The supplier's unit selling price
c_R	The retailer's unit cost
c_S	The supplier's unit cost
g_R	The retailer's understock cost
g_S	The supplier's understock cost
v_R	The retailer's overstock cost
D	Demand, D~N(μ, σ)
i_ϵ	The supplier's risk-free market investment rate
R_R	The retailer's limited working capital
R_S	The supplier's available working capital used to invest ($)
π_X	The X's profit ($X = R, S, SC$)
Q_X^*	The X's optimal order quantity ($X = R, S, SC$)
η_X	The efficiency of X ($X = R, S, SC$)
i_R	Loan rate charged on the capital constrained retailer by the supplier
${\pi_{R:\,supplier\,-led}}^*$	Optimal retailer profit in supplier led Stackelberg game
${\pi_{R:\,retailer-led}}^*$	Optimal retailer profit in retailer led Stackelberg game
${\pi_{S:\,supplier\,-led}}^*$	Optimal supplier profit in supplier led Stackelberg game
${\pi_{S:\,retailer-led}}^*$	Optimal supplier profit in retailer led Stackelberg game
${\pi_{SC:\,supplier\,-led}}^*$	Optimal supply chain profit in supplier led Stackelberg game
${\pi_{SC:\,retailer-led}}^*$	Optimal supply chain profit in retailer led Stackelberg game
w	The discounted wholesale price in modified wholesale price contract
i	The discounted loan rate in the modified wholesale price contract
λ	The contract parameter
T_1	Transfer payment made in retailer-led coordination
T_2	Transfer payment made in supplier-led coordination

3.2.2 Financing Profit Models

The retailer and supplier's profits are typical profit models with the added loan interest and principal paid by the retailer to the supplier. The liquid supplier is assumed to have enough available capital R_S to lend to the retailer. The retailer has limited working capital, R_R, and can only afford to purchase α (0<α<1) portion of the desired order quantity and requests to borrow enough to cover the remainder of the supplier so the supplier lends, β $(0 < \beta < 1)$, portion of his total available capital R_S to retailer. Therefore,

$$\alpha = \frac{R_R}{Q * c_R} \tag{3.24}$$

The retailer requests to borrow enough to cover the remainder from the supplier, therefore

$$\beta = \frac{Q * c_R - R_R}{R_S} \tag{3.25}$$

The supplier chooses to invest the rest of the capital (1- β) R_S to the market at the rate i_ϵ, which is exogenously specified and assumed to be greater than or equal to i_R, similar to [96]. The supply chain profit is the summation of both the retailer and the supplier profits. The initial profit models and corresponding optimal order quantities for the retailer, supplier and supply chain (Q_R^*, Q_S^*, Q_{SC}^*), are shown in Equations 3.26-3.31, respectively.

$$\pi_R = p_R \min(Q, D) + v_R \max(Q - D, 0) - g_R \max(D - Q, 0) - (1 - \alpha) Q c_R i_R - c_R Q \tag{3.26}$$

$$\pi_S = (p_S - c_S)Q - (i_\epsilon - i_R) R_S \beta - g_S \max(D - Q, 0) + (1 - \beta) R_S i_\epsilon \tag{3.27}$$

$$\pi_{SC} = \pi_R(Q, i_R, p_S) + \pi_S(Q, i_R, p_S) \tag{3.28}$$

$$Q_R^* = F^{-1}\left(\frac{p_R + g_R - c_R - c_R i_R}{p_R + g_R - v_R}\right) \tag{3.29}$$

$$Q_S^* = F^{-1}\left(\frac{(p_S - c_S) + (i_R - 2 i_\epsilon) c_R + g_S}{g_S}\right) \tag{3.30}$$

$$Q_{SC}^{*} = F^{-1}\left(\frac{p_R+(p_S-c_S)+g_R+g_S-(1+2i_\epsilon)c_R}{p_R+g_R+g_S-v_R}\right) \tag{3.31}$$

LEMMA 7. ***Q_R^* exists and is unique when $i_R < \frac{2(p_R-c_R)}{c_R}$.***

Lemma 7 shows that there is an upper bound on the interest rate that will guarantee existence of the retailer's optimal order quantity (see the proof in Appendix 3).

LEMMA 8. ***The unique Q_S^* exists only when $(2i_\epsilon - i_R)$ is bounded by one and two profit percentages: $(\frac{p_S-c_S}{c_R} < 2i_\epsilon - i_R < \frac{2(p_S-c_S)}{c_R})$. When $(2i_\epsilon - i_R)$ is less than the profit percentage $\frac{p_S-c_S}{c_R}$, the supplier's profit has no extreme value but is monotonically increasing in order quantity Q.***

Lemma 8 demonstrates that Q_S^* only exists under one condition $(\frac{p_S-c_S}{c_R} < 2i_\epsilon - i_R)$. As the retailer's loan rate, i_R approaches i_ϵ, the profit percentage $\frac{p_S-c_S}{c_R}$ would be less than i_ϵ. This, in practice would not occur as any supplier would aim to make more than the risk-free interest rate by being in business. Therefore, Q_{SC}^* is the optimal order quantity for the supplier to optimize profit (see the proof in Appendix 4).

LEMMA 9. ***Q_{SC}^* exists and has a unique value when the lower bound of the market invest rate (i_ϵ) equals half of the summation of the profit percentage and retailer's loss percentage ($i_\epsilon > \frac{p_S-c_S}{2c_R} + \frac{v_R-c_R}{2c_R}$) while the upper bound is the summation of the retailer's profit percentage and the integrated profit percentage ($i_\epsilon \le \frac{p_R-c_R}{c_R} + \frac{p_S-c_S}{c_R}$).***

Lemma 9 shows that the supply chain optimal order quantity exists, thus there exist two optimal order quantities (Q_R^* and Q_{SC}^*) in the supply chain system. The former (Q_R^*) optimizes the retailer and the latter (Q_{SC}^*) optimizes the supplier and supply chain (see the proof in Appendix 5).

It is assumed here, however, that the retailer has control over the order quantity, therefore will aim to order Q_R^*. This however, makes for a suboptimal supply chain profit as shown in [12], and in order to coordinate the supply chain and optimize overall supply chain profits, the retailer should order Q_{SC}^*. However, this will yield a lower profit for the retailer. Thus, an efficiency model is introduced to track the operational efficiency of each party.

3.2.3 Efficiency Models

An order quantity of Q_R^* will optimize retailer profit and Q_{SC}^* will optimize the supplier and supply chain profit as indicated in Section 3.2.2. Efficiency is generally calculated as the ratio of actual profit to optimal profit. Both retailer-led and supplier-led Stackelberg game is studied and compared among Traditional Newsvendor Model with Financing (TNMF), coordinated supply chain with Modified Wholesale Price Contract (MWPC), and coordinated supply chain with Traditional Wholesale Price Contract (TWPC). Profits of the three parties will be analyzed against the optimal, thus efficiencies, η, are defined in Equations 3.32-3.34, where subscript –A denotes the particular case studied.

$$\eta_R = \frac{\pi_{R-A}}{\pi_R(Q_R^*)} \tag{3.32}$$

$$\eta_S = \frac{\pi_{S-A}}{\pi_S(Q_S^*)} \tag{3.33}$$

$$\eta_{SC} = \frac{\pi_{SC-A}}{\pi_{SC}(Q_{SC}^*)} \tag{3.34}$$

3.2.4 Two-Echelon Supply Chain Financing Coordination

MWPC is introduced to coordinate two-echelon supply chain financing. Both supplier-led and retailer-led Stackelberg game is applied to explore the profit and efficiency equilibrium in different scenarios, which are explored in Section 3.2.4.1 and 3.2.4.2 respectively. A numerical example is followed in Section 4.2 to verify the conclusions generated in this part.

3.2.4.1 Stackelberg Game in TNMF

In this section, a general Stackelberg game is applied to explore the optimal wholesale price, loan rate and order quantity in TNMF, as well as the optimal profits and efficiencies of supply chain entities. Some interesting findings toward how the profit and efficiency depend on wholesale price and order quantity will be presented.

As shown in Section 3.2.2, the retailer's optimal order quantity Q_R^* is a function of wholesale price p_S and loan rate i_R, which are controlled by the supplier. Hence, the operation decisions of the supply chain are changing with the value of the wholesale price and loan rate. Here, two variations of the classical Stackelberg game are formulated: retailer-led and supplier-led. There are three parameters involved in the game negotiation: wholesale price, loan rate and order quantity, among which, wholesale price and loan rate are the supplier's decision variables while order quantity is the retailer's decision variable.

3.2.4.1.1 Retailer-led Stackelberg in TNMF (RS-TNMF)

In the retailer-led game, the retailer (leader) will make the first move and set the values of the optimal order quantity, Q^* being aware of the supplier's (follower) wholesale price, $p_S^*(Q)$, and loan rate, $i_R^*(Q)$. The supplier's decision of loan rate and wholesale price is made based on maximizing profit using Equation 3.27:

$$p_S^*(Q) = \frac{R_R}{Q} \tag{3.35}$$

$$i_R^*(Q) = \frac{1}{Q}\int_Q^{+\infty}(x-Q)f(x)dx + 2i_\epsilon - 1 \tag{3.36}$$

Using backwards induction, the retailer's best strategy with an optimal order quantity, Q^*, is found by substituting Equations 3.35 and 3.36 into Equation 3.26, and optimizing, shown in Equation 3.37.

$$p_R - g_R - (p_R + g_R - v_R)F(Q^*) + \frac{R_R}{Q^{*2}}\left(\int_0^{+\infty}|x - Q^*|f(x)dx\right) = 0 \tag{3.37}$$

The corresponding optimal profits for retailer, supplier, and SC in retailer-led game are:

$$\pi_{R:\,supplier-led}{}^* = p_R \min(Q^*, D) + v_R((p_S^*(Q^*)) \max(Q^* - D, 0) - g_R(p_S^*(Q^*)) \max(D - Q^*, 0) - Q^* c_R(p_S^*(Q^*)) i_R^* + i_R^* R_R - Q^* c_R(p_S^*(Q^*)) \tag{3.38}$$

$$\pi_{S:\,supplier-led}{}^* = (p_S^*(Q^*) - c_S)Q^* - (i_\epsilon - i_R^*)(Q^* c_R(p_S^*(Q^*)) - R_R) - g_S(p_S^*(Q^*)) \max(D - Q^*, 0) + (R_S + R_R - Q^* c_R(p_S^*(Q^*)))i_\epsilon \tag{3.39}$$

$$\pi_{SC:\,supplier-led}{}^* = \pi_{R:\,supplier-led}{}^* + \pi_{S:\,supplier-led}{}^* \tag{3.40}$$

Corollary 10. ***In the RS-TNMF, the risk-neutral supplier's response on loan rate is independent of the order quantity but increasing in the supplier's market investment rate.***

The first term in Equation 3.36 approaches zero and thus, only the second term is used to decide the optimal loan rate. Therefore, the supplier's optimal loan rate is independent of the retailer's order quantity, but dependent on the market reinvestment rate in RS-TNMF. The optimal loan rate is only non-negative for market reinvestment rates greater than 50%, which is unrealistic given the supplier is risk-neutral. In this scenario, with the optimal wholesale price given, the profit of both the retailer and supplier are independent of the loan rate.

> **Corollary 11.** ***In the RS-TNMF game, the overall supply chain profit and efficiency can be maximized by the wholesale price, however, long-run profits and efficiencies of the retailer and supplier are inversely related.***

When Q approaches zero, $\frac{d\pi_{SC}}{dQ} < 0$ while $\frac{d\pi_{SC}}{dQ} > 0$ when Q approaches μ. Therefore, there exists the Q $\in (0, \mu)$ that leads to the minimum $\pi_{SC:\,supplier-led}{}^{*}$. Given the wholesale price has a strictly inverse relation with Q (see Equation 3.35), there exists the wholesale price which results in maximum supply chain profit. Likewise, for large enough Q, $\frac{d\pi_S}{dQ} \approx -c_S < 0$, indicating the supplier profit decreases with Q or increases with wholesale price. According to the definition of efficiency in Section 3.2.3, the changing trend of one entity's efficiency keeps consistent with the entity's profit. The similar proving process applies to the retailer, which concludes the long-run retailer profit and efficiency decrease with wholesale price.

3.2.4.1.2 Supplier-led Stackelberg in TNMF (SS-TNMF)

In the supplier-led game, the supplier (leader) has the priority to decide the wholesale price and loan rate first, anticipating the order quantity the retailer (follower) will respond with. Using backward induction, the supplier's optimal strategy is based on the retailer's optimal response shown in Equation 3.41:

$$Q^*(i_R, p_S) = F^{-1}\left(\frac{p_R+g_R-p_S-p_S i_R}{p_R+g_R-v_R}\right) \tag{3.41}$$

Substitute Q^*into Eq. 3.27, the updated supplier profit is obtained. By partial differentials, supplier's optimal strategy with ${p_S}^*$ and ${i_R}^*$ as a result of the retailer's optimal order quantity $Q^*(i_R, p_S)$ satisfy,

$$\begin{cases} (p_S - c_S + g_S - g_S F(Q) - p_S i_\epsilon) Q'(i_R) + Q(i_R) p_S - R_R = 0 \\ (p_S - c_S + g_S - g_S F(Q) - 2p_S i_\epsilon + i_R p_S) Q'(p_S) + (1 - 2i_\epsilon + i_R) Q(p_S) - \int_Q^{+\infty} (x - Q) f(x) dx = 0 \end{cases} \tag{3.42}$$

The corresponding optimal profits for retailer, supplier, and SC under SS-TNMF are:

$$\pi_{R:\,retailer-led}{}^* = p_R \min(Q^*({i_R}^*, {p_S}^*), D) + v_R({p_S}^*) \max(Q^*({i_R}^*, {p_S}^*) - D, 0) - g_R({p_S}^*) \max(D - Q^*({i_R}^*, {p_S}^*), 0) - Q^*({i_R}^*, {p_S}^*) i_R^* c_R({p_S}^*) + i_R^* R_R - c_R({p_S}^*) Q^*({i_R}^*, {p_S}^*) \tag{3.43}$$

$$\pi_{S:\,retailer-led}{}^* = ({p_S}^* - c_S) Q^*({i_R}^*, {p_S}^*) - (i_\epsilon - i_R^*)(Q^*({i_R}^*, {p_S}^*) c_R({p_S}^*) - R_R) - g_S({p_S}^*) \max(D - Q^*({i_R}^*, {p_S}^*), 0) + (R_S + R_R - Q^*({i_R}^*, {p_S}^*) c_R({p_S}^*)) i_\epsilon \tag{3.44}$$

$$\pi_{SC:\,retailer-led}{}^* = \pi_{R:\,retailer-led}{}^* + \pi_{S:\,retailer-led}{}^* \tag{3.45}$$

Corollary 12. ***In SS-TNMF game, the profit and efficiency of the retailer and supply chain are optimized in order quantity, while the supplier's profit and efficiency are monotonically increasing.***

In SS-TNMF, there is no stable Stackelberg equilibrium because the supplier has the incentive to drive up the loan rate and wholesale price to its maximum. However, given the retailer would choose to switch to other suppliers if the charged wholesale price and loan rate are incredibly high, there does in practice exist an upper limit for the supplier's decision variables. With the set values for loan rate and wholesale price, the retailer profit would increase first since the increased order quantity will decrease goodwill costs. Once it reaches the optimal order quantity, the retailer profit starts decreasing due to overstock. Increasing the order quantity will increase the supplier's sales profit, which can be easily verified through optimizing supplier's profit with the order quantity as the independent variable.

3.2.4.2 Stackelberg Game with MWPC

To improve the overall supply chain performance, wholesale price contract [12] is employed to coordinate the supply chain. Wholesale price contract is chosen due to the goal of encouraging the capital constrained retailer to order more, while simultaneously lowering the wholesale price and loan rate to benefit both of the two entities. The traditional wholesale price contract is modified to incorporate financing term, loan rate, of which the mathematical form is:

$$w = \lambda_1 \, p_S \tag{3.46}$$

$$i = \lambda_2 i_R \tag{3.47}$$

Where p_S, i_R are the initial wholesale price and loan rate set by the supplier, λ_1 and λ_2 $(0 < \lambda_1, \lambda_2 \leq 1)$ are contract parameters for wholesale price and loan rate, respectively. $\lambda_1$and λ_2 can equal different values, depending on the supply chain coordination contract design. To simplify complex mathematical calculation, we set $\lambda_1 = \lambda_2 = \lambda$, which reaches the same coordination effect with $\lambda_1 \neq \lambda_2$. $(1 - \lambda)$ is the discount rate applied to reduce the wholesale price and loan rate. With applying modified wholesale price contract, there exists the common decision variable, λ for both retailer and supplier to decide their optimal strategies.

Equation 3.46 and 3.47 are constructed based on the definition of wholesale price contract in supply chain coordination [12]. The contract is modified by extending its application to loan rate, and is also combined with the model assumptions to the specific contract formulation.

Discounts are applied to the wholesale price and loan rate at the same time, which differs from the previous research on employing the coordination mechanisms. The proposed contract is compared with the traditional wholesale price contract that only applies discount to the wholesale price, the numerical study of which is shown in Section 4.2.

3.2.4.2.1 Retailer-led Stackelberg with MWPC (RS-MWPC)

Similar to explore RS-TNMF using backward induction, the retailer's optimal strategy in RS-MWPC is calculated, being aware of the supplier's response. Substituting Equations 3.46-3.47 into supplier's profit function shown in Equation 3.27, solving for λ_S^*,

$$\lambda_S^* = \frac{p_S \int_Q^{+\infty}(x-Q)f(x)dx + 2Qi_\epsilon p_S + R_R i_R - Qp_S}{2Qi_R p_S} \tag{3.48}$$

Where λ_S^* represents the optimal value for contract parameter, λ, in RS-MWPC and the supplier's optimal response (w_{s-led}^*, i_{s-led}^*) is found as:

$$w_{s-led}^* = p_S \lambda_S^* \tag{3.49}$$

$$i_{s-led}^* = i_R \lambda_S^* \tag{3.50}$$

Substituting Equations 3.49 and 3.50 into the retailer's profit function shown in Equation 3.26, the retailer's optimal strategy Q^* (λ_S^*) can be obtained through maximizing the retailer's profit function, $\pi_R(Q^*, w_{s-led}^*, i_{s-led}^*)$

Similar to Section 3.2.4.1.1, the optimal profits of the retailer, supplier, and supply chain under RS-MWPC are also obtained through substituting w_{s-led}^*, i_{s-led}^*, and Q^* (λ_S^*) into their profit functions, respectively.

$$\pi_{R:\,supplier-led}{}^{*} = p_R \min(Q^*\,(\lambda_S^*)\,, D) + v_R(w_{s-led}^*) \max(Q^*\,(\lambda_S^*)\,-D, 0) - g_R(w_{s-led}^*) \max(D - Q^*\,(\lambda_S^*)\,, 0) - Q^*\,(\lambda_S^*)\,c_R(w_{s-led}^*) i_{s-led}^* + i_{s-led}^* R_R - Q^*\,(\lambda_S^*)\,c_R(w_{s-led}^*) - T_1 \quad (3.51)$$

$$\pi_{S:\,supplier-led}{}^{*} = (w_{s-led}^* - c_S) Q^*\,(\lambda_S^*) - (i_\epsilon - i_{s-led}^*)(Q^*\,(\lambda_S^*) c_R(w_{s-led}^*) - R_R) - g_S(w_{s-led}^*) \max(D - Q^*\,(\lambda_S^*), 0) + (R_S + R_R - Q^*\,(\lambda_S^*) c_R(w_{s-led}^*)) i_\epsilon + T_1 \quad (3.52)$$

$$\pi_{SC:\,supplier-led}{}^{*} = \pi_{R:\,supplier-led}{}^{*} + \pi_{S:\,supplier-led}{}^{*} \quad (3.53)$$

To better split the increased supply chain profit between the two parties, a dedicated transfer,T_1 from the retailer to the supplier is necessary. The optimal value of T_1 is determined through optimizing overall supply chain performance, increasing profits and efficiencies for both the retailer and supplier.

3.2.4.2.2 Supplier-led Stackelberg with MWPC (SS-MWPC)

Similar to SS-TNMF, substituting Equations 3.46-3.47 into the retailer's profit function, which is then optimized to obtain the retailer's optimal response strategy, $Q^*\,(\lambda_R^*)$ where λ_R^* is taken as the retailer's optimal contract parameter value:

$$Q^*\,(\lambda_R^*) = F^{-1}\left(\frac{p_S i_R {\lambda_R^*}^2 + 2p_S\lambda_R^* - 2p_R - 2}{2p_S\lambda_R^* - 2p_R + 14}\right) \quad (3.54)$$

Subsequently, the supplier's optimal strategy (w_{r-led}^*, i_{r-led}^*) in the SS-MWPC is calculated:

$$w_{r-led}^* = p_S \lambda_R^* \quad (3.55)$$

$$i_{r-led}^* = i_R \lambda_R^* \quad (3.56)$$

The optimal profits of the retailer, supplier, and supply chain under SS-MWPC, see Equations 3.57-3.59, are obtained through substituting w^*_{r-led}, i^*_{r-led}, and $Q^*\ (\lambda^*_R)$ into Equations 3.26-3.28, respectively. A dedicated transfer, T_2 in SS-MWPC is necessary for splitting the overall increased supply chain profit. A numerical example on T_1 and T_2 is studied in Section 4.2.

$$\pi_{R:\,retailer-led}{}^* = p_R \min(Q^*\ (\lambda^*_R)\ , D) + v_R(w^*_{r-led}) \max(Q^*\ (\lambda^*_R) - D, 0) - g_R(w^*_{r-led})\max(D - Q^*\ (\lambda^*_R), 0) - Q^*\ (\lambda^*_R)\ i^*_{r-led} c_R(w^*_{r-led}) + i^*_{r-led} R_R - c_R(w^*_{r-led}) Q^*\ (\lambda^*_R)\ - T_2 \tag{3.57}$$

$$\pi_{S:\,retailer-led}{}^* = (w^*_{r-led} - c_S) Q^*\ (\lambda^*_R)\ - (i_\epsilon - i^*_{r-led})(Q^*\ (\lambda^*_R) c_R(w^*_{r-led}) - R_R) - g_S(w^*_{r-led}) \max(D - Q^*\ (\lambda^*_R)\ , 0) + (\ R_S + R_R - Q^*\ (\lambda^*_R)\ c_R(w^*_{r-led})) i_\epsilon + T_2 \tag{3.58}$$

$$\pi_{SC:\,retailer-led}{}^* = \ \pi_{R:\,retailer-led}{}^* + \pi_{S:\,retailer-led}{}^* \tag{3.59}$$

Corollary 13. ***In both RS-MWPC and SS-MWPC, the retailer's profit increases in the discount rate while the supplier's profit decreases in the discount rate. Both of the retailer and supplier can benefit from the MWPC as long as the discount rate is appropriately set.***

The higher the discount rates, the lower the wholesale price and loan rate. Thus, the retailer has an increasing profit while the supplier's profit is decreasing in the discount rate. Due to the complexity of the formula, a numerical example shown in Section 4.2 is used to demonstrate the interval of the contract parameter where both the retailer and supplier are better off from utilizing the proposed contract, MWPC.

3.3 Coordinating the Multi-Echelon, Multi-Period SCF

We consider a multi-period supply chain with multiple risk-neutral firms, where the downstream retailer is capital constrained facing stochastic market demand. To meet market demand, the retailer needs financing and opts to borrow from the neighboring upstream partner at a certain rate, i. The retailer's neighboring upstream partner is assumed to be liquid and available to support the capital-constrained retailer across the financing period. Each period, the retailer aims to order the optimal order quantity as a function of its costs and market demand. The benchmark to determine whether or not the retailer needs to finance in each period as well as the financing amount depends on the difference between the purchasing cost of the optimal order quantity in this period and the retailer's net accumulated profits made in previous periods. If the difference is positive, then the retailer will have to finance from his neighboring upstream partner and the financing amount should be no less than the difference. Otherwise, there is no need for the retailer to finance as they are no longer capital constrained in this period. The retailer's financing amount in one specific period will be paid back as installments starting in the next period. The objectives of the supply chain entities are to maximize their own profit and efficiency under the premise of maximizing average supply chain profit across multiple periods. The research objective is to explore the optimal contract periods as well as the coordination mechanism, through which the various supply chain individuals will work together to produce better overall supply chain profit.

3.3.1 Assumptions and Notations

A serial supply chain is employed consisting of N stages with M financing entities ($M < N$) and ($P \leq M$) financing receivers. In this research, $N > 2$ and $M = P = 1$(see Figure 3-2). The physical flow of material each period is from upstream to downstream while the financial flow is in the other direction. Entity m ($m \in P$) is capital-constrained and needs to finance from entity $k(k \in M)$, where R_t is noted as the financing amount in period t.

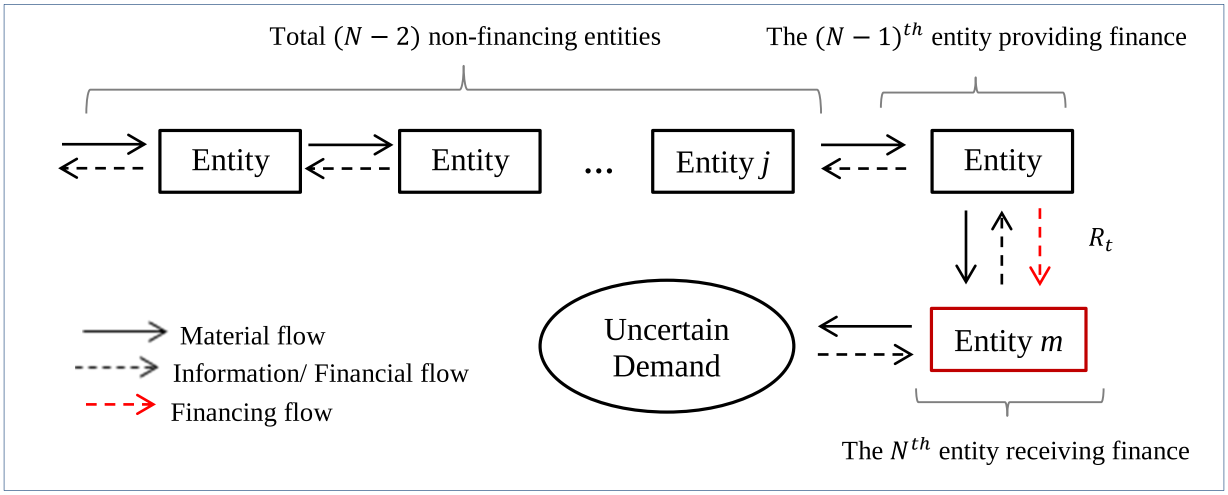

Figure 3-2: Multi-Echelon SCF Model

Figure 3-3 demonstrates the financing process between entity k and m in the perspective of the financing provider, where R_t .is regarded as cash outflow for financing provider at the beginning of period t and noted by the downward solid arrows. The capital constrained entity m will pay installment amounts (annuities), noted as A_t (corresponding with the amount borrowed, R_t), to entity k at the end of the period until T. A_t is regarded as the forthcoming cash inflow and noted in upward dashed arrow.

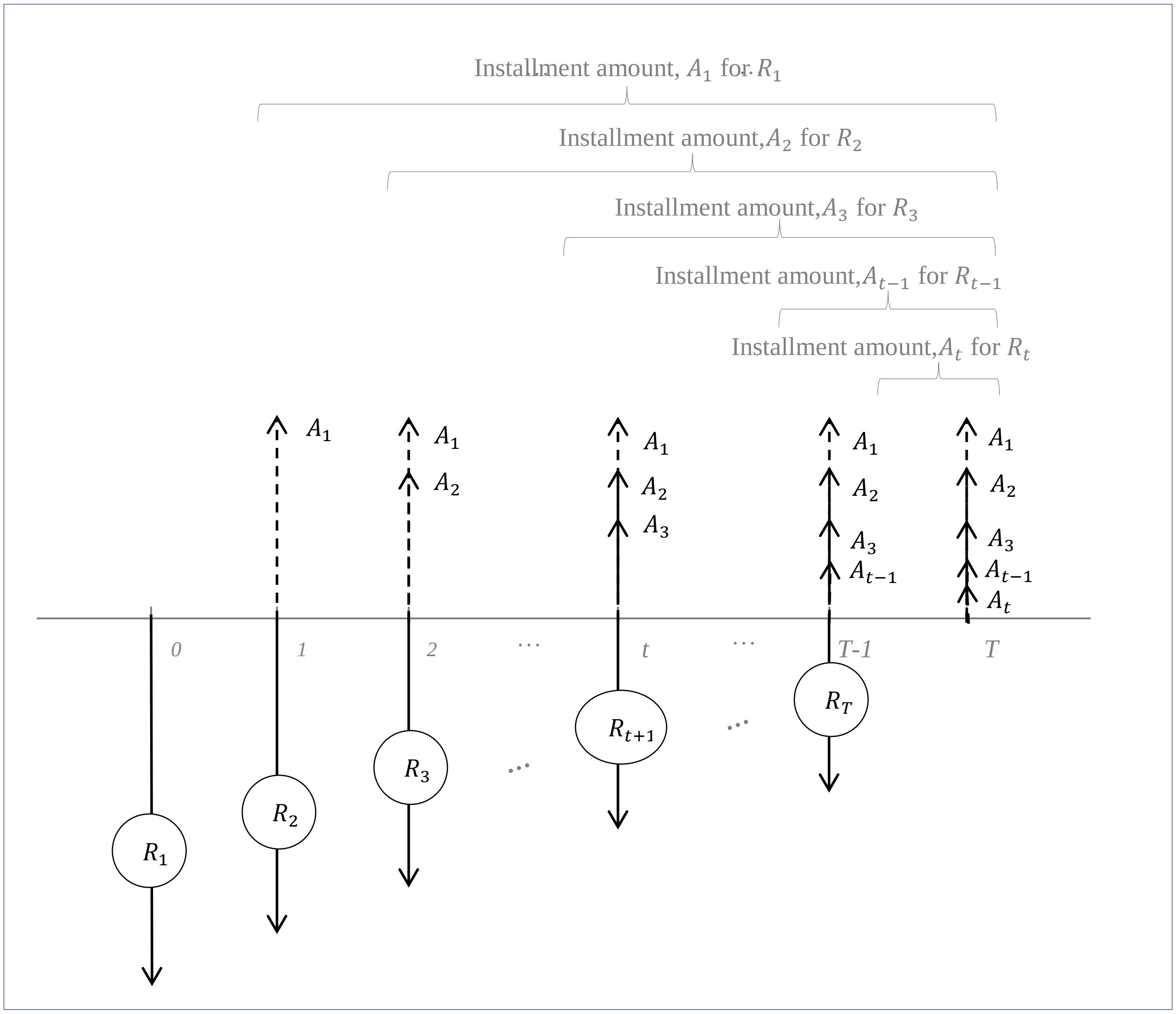

Figure 3-3:Multi-period Supply Chain Financing from the Provider's Perspective

The model assumptions are listed below:

1. Lead time is assumed to be negligible.
2. The demand and cost information is shared completely between the supplier and the retailer.
3. The retailer will not default on the loan obligation, and the sale price is exogenously specified.

4. Financing provider is liquid and the market investment rate is determined exogenously.

5. Each supply chain entity's unit purchasing cost, retailing price, transportation cost, holding cost, back order cost stays consistent across all periods.

6. The customer demand in each period is assumed to be independent of the fixed market selling price and seasonality and follows a normal distribution with known parameter values.

7. The available working capital of the capital constrained retailer is assumed to be zero at the beginning of the first period; Initial inventory for all of the supply chain entities at the beginning of the first period are assumed to be zero.

8. The supply chain entity related unit purchase cost, transportation cost, holding cost, backorder cost, and retail price is assumed to be fixed in each period.

The demand uncertainty leads to potential overstock and/or understock conditions. Unfilled orders at the end of each period are backordered and overstock is held for the next period. All supply chain entities seek to satisfy an order fill rate, s. The retailer's liquid upstream party can invest working capital in the market at rate ε or lend to the retailer, if needed, at rate i. The opportunity cost is incorporated into the liquid upstream party's profit model, and becomes negative when the loan rate charged onto downstream retailer is less than the general market investment rate. The model parameters are given in Table 3-3.

Table 3-3: Parameters and Variables: Multi-Echelon SCF

T	Number of contract periods studied
N	Number of supply chain entities
s	Supply chain entity required order fill rate
i	Loan rate across T periods
ε	General market investment rate
t	The specific period, $\mathrm{t} = 1, 2, \ldots, \mathrm{T}$
j	The supply chain entities that do not involve in financing $(1 \leq j \leq N-2)$
k	The supply chain entity that provides financing $(k = N-1)$
m	The supply chain entity that receives financing $(m = N)$
Q_{xt}	Supply chain entity x's order quantity in period t $(x = j, k, m)$
Q'_{xt}	Supply chain entity x's order quantity through maximizing x's profit
c_x	Supply chain entity x's purchase cost per unit $(x = j, k, m)$
p_x	Supply chain entity x's retail price per unit $(x = j, k, m)$
t_x	Supply chain entity x's transportation cost per unit $(x = j, k, m)$
h_x	Supply chain entity x's holding cost per unit $(x = j, k, m)$
b_x	Supply chain entity x's backorder cost per unit $(x = j, k, m)$
I_{xt}	Supply chain entity x's inventory left at end of period t $(x = j, k, m)$
D_t	Market demand in period $t, D_t \sim N(\mu, \sigma)$
L	Capacity of entity 1 per period
R_t	Actual financing amount in period t
π_{xT}	Supply chain entity x's average profit made across T periods $(x = j, k, m)$
π_{xT-c}	Supply chain entity x's average profit made in centralized case
π_{xT-d}	Supply chain entity x's average profit made in decentralized case
π_{scT}	The average supply chain profit made across T periods

Table 3-3: Parameters and Variables: Multi-Echelon SCF (continued)

π_{scT-c}	The average supply chain profit made in centralized case across T periods
π_{scT-d}	The average supply chain profit made in decentralized case across T periods
η_{xT}	Supply chain entity $x's$ efficiency across T periods ($x = j, k, m$)
η_{xT-c}	Supply chain entity $x's$ efficiency in centralized case
η_{xT-d}	Supply chain entity $x's$ efficiency in decentralized case
η_{scT}	Overall supply chain efficiency across T periods
η_{scT-d}	Overall supply chain efficiency in decentralized case
i_{-c}	Loan rate in centralized case across T periods
i_{-d}	Loan rate in decentralized case across T periods
T_{sc}^{*}	The supply chain's optimal number of contract periods
T_{x}^{*}	The entity $x's$ optimal number of contract periods ($x = j, k, m$)
$\theta_{j\to j+1}$	The revenue share portion from entity j to $j+1$ across T periods
$r_{1-\mathrm{I}}$	The discount rate of decentralized loan rate in RSLRD coordination
$r_{1-\mathrm{II}}$	The discount rate of decentralized loan rate in RSRWPC coordination
r_{2}	The coordinated discount rate of financing provider's retail price

3.3.2 Financing Profit and Efficiency Models

The supply chain profit and efficiency models are developed for T periods. We focus on optimizing the average supply chain profits over T periods instead of the total supply chain profits. Since demand is stochastic, this optimized average supply chain profit will change with the uncertain market demands. The average profit models for all supply chain entities and the supply chain are given in Equations 3.60-3.64.

3.3.2.1 Profit Model for Entities Not Involved in Financing

For entity $j \in J = (N - M - P)$ that is not involved with financing, the profit components include sales revenue, purchase cost, transportation cost, holding cost, and backorder cost. Over T periods, supply chain entity $j's$ average profit model is:

$$\pi_{jT}(Q_{jt}) = \frac{1}{T}\sum_{t=1}^{T}[(p_j - c_j)Q_{(j+1)t} - t_j MIN(Q_{jt} + I_{j(t-1)}, Q_{(j+1)t}) - h_j(Q_{jt} + I_{j(t-1)} - Q_{(j+1)t})^{+} - b_j(Q_{(j+1)t} - Q_{jt} - I_{j(t-1)})^{+}] \qquad (3.60)$$

3.3.2.2 Profit Model for the Entity Which Provides Financing

For the entity $k \in M$ that provides financing to capital constrained retailer , the profit components include sales revenue, principal and interest collected from loan, purchase cost, transportation cost, holding cost, backorder cost, and opportunity cost. Thus, supply chain entity $k's$ average profit model over T periods is:

$$\pi_{kT}(Q_{kt}, i) = \frac{1}{T}\sum_{t=1}^{T}\Big[(p_k - c_k)Q_{(k+1)t} + \sum_1^t R_t(A/P, i, T-t+1) - t_k MIN(Q_{kt} + I_{k(t-1)}, Q_{(k+1)t}) - h_k(Q_{kt} + I_{k(t-1)} - Q_{(k+1)t})^{+} - b_k(Q_{(k+1)t} - Q_{kt} - I_{k(t-1)})^{+} - \sum_1^t R_t((A/P, \varepsilon, T-t+1) - (A/P, i, T-t+1)) - R_t\Big] \qquad (3.61)$$

3.3.2.3 Profit Model for the Entity Which Receives Financing

For the entity $m \in P$ that requires financing, the profit components include sales revenue, financing amount received, purchase cost, transportation cost, holding cost, backorder cost, and paid interest. Thus, supply chain entity $m's$ average profit model over T periods is:

$$\pi_{mT}(Q_{mt}, i) = \frac{1}{T}\sum_{t=1}^{T}\Big[(p_m - c_m)D_t + R_t - \sum_1^t R_t(A/P, i, T-t+1) - t_m MIN(Q_{mt} + I_{m(t-1)}, Q_{(m+1)t}) - h_m(Q_{mt} + I_{m(t-1)} - Q_{(m+1)t})^{+} - b_m(Q_{(m+1)t} - Q_{mt} - I_{m(t-1)})^{+}\Big] \qquad (3.62)$$

3.3.2.4 Average Supply Chain Profit over T Periods

The average supply chain profit is equal to the summation of all of the supply chain entities' average profits over T periods:

$$\pi_{scT}\left(Q_{jt}, Q_{kt}, Q_{mt}, i\right) = \pi_{jT}\left(Q_{jt}\right) + \pi_{kT}(Q_{kt}, i) + \pi_{mT}(Q_{mt}, i) \tag{3.63}$$

Applying Equations 3.60-3.63, the mathematical model to optimize the average supply chain profit over T periods is demonstrated in Equations 3.64-3.70. Decision variables are Q_{jt}, Q_{kt}, Q_{mt}, and $\boldsymbol{i}$ noted in Table 3-3.

$$\text{MAX} \quad \pi_{scT-c}\left(Q_{jt}, Q_{kt}, Q_{mt}, i\right) \tag{3.64}$$

$$\text{s.t. } MIN\left(Q_{jt} + I_{j(t-1)}, Q_{(j+1)t}\right) \geq s * Q_{(j+1)t}, \quad \forall j \in J, \forall t \in T \tag{3.65}$$

$$MIN\left(Q_{kt} + I_{k(t-1)}, Q_{(k+1)t}\right) \geq s * Q_{(k+1)t}, \quad \forall k \in M, \forall t \in T \tag{3.66}$$

$$MIN\left(Q_{mt} + I_{m(t-1)}, Q_{(m+1)t}\right) \geq s * Q_{(m+1)t}, \quad \forall m \in P, \forall t \in T \tag{3.67}$$

$$MAX\left(Q_t + I_{1(t-1)}, Q_{2t}\right) \leq L + I_{1(t-1)}, \quad \forall t \in T \tag{3.68}$$

$$T > 2 \tag{3.69}$$

$$0 < i < 1 \tag{3.70}$$

Equations 3.65-3.67 are built for fill rate constraints, where the supply chain entities have to satisfy the minimum order fill rate. Equation 3.68 is for the capacity constraint for first entity. The solution to this model will yield the variables described in Table 3-3, from which the optimal contract length, T_{sc}^{*}, and optimal profits of the supply chain and all the entities, $\pi^{*}{}_{scT}$, π_{jT}, π_{kT}, and π_{mT} can be obtained.

Efficiency is an important parameter to measure the degree of profitability in different supply chain settings. The optimal profit in T periods, for supplier chain members, $j, m,$ and k are found by optimizing Equations 3.60-3.62 (${\pi^*}_{jT}, {\pi^*}_{kT}, {\pi^*}_{mT}$) and the actual profit is the profit yielded from the supply chain maximization resulting from solving the mathematical model built in Equations 3.64-3.70 ($\pi_{jT}, \pi_{kT}, \pi_{mT}$). In this study, the entity's T-period efficiency is calculated as the ratio of actual average profit to the optimal average profit across T periods, shown in Equations 3.71-3.73.

$$\eta_{jT} = \frac{\pi_{jT}}{\pi^*_{jT}} \tag{3.71}$$

$$\eta_{kT} = \frac{\pi_{kT}}{\pi^*_{kT}} \tag{3.72}$$

$$\eta_{mT} = \frac{\pi_{mT}}{\pi^*_{mT}} \tag{3.73}$$

3.3.3 Multi-Echelon Supply Chain Coordination

The supply chain individual's efficiency varies under different T periods. Therefore, each supply chain entity *j, k, m* will have an optimal contract period, T_j^* , T_k^* , T_m^* respectively which may not be consistent with the supply chain's optimal contract period, T_{sc}^* . When T_j^*, T_k^* , $T_m^* \neq T_{sc}^*$, T_{sc}^* should be given priority from the perspective of supply chain profit and efficiency maximization. However, choosing T_{sc}^* may harm some individual's profit and efficiency optimization while benefitting others, which causes conflicts within the SC and is detrimental to the efficiency of the entire chain. Therefore, proper coordination mechanisms are necessary to be implemented to integrate supply chain actions.

If there is no coordination mechanism implemented, supply chain entities will make decisions based on maximizing their own interests under T_{sc}^{*}. For entity $m \in P$, $Q'_{mt}(t = 1,2, \ldots T_{sc}^{*})$ is determined through maximizing π_{mT} shown in Equation 3.62. For entity $k \in M$, $Q'_{kt}(t = 1,2, \ldots T_{sc}^{*})$ and i' are obtained resulting from maximizing π_{kT} shown in Equation 3.61. For entity $j \in J$, $Q'_{jt}(t = 1,2, \ldots T_{sc}^{*})$ is determined through maximizing π_{jT} shown in Equation 3.60. Thus, the suboptimal average SC profit across T_{sc}^{*} is,

$$\pi_{scT-d}(Q'_{mt}, Q'_{kt}, i', Q'_{jt}) = \pi_{mT-d} + \pi_{kT-d} + \pi_{jT-d} \quad (3.74)$$

3.3.3.1 Revenue Sharing Combing Revised Wholesale Price Contract (RSRWPC)

The proposed coordination mechanism $(\theta_{j\to j+1}, r_{1-\mathrm{II}}, r_2)$ combines revenue sharing and a revised wholesale price contract. Pairwise revenue sharing is implemented between all pairs of adjacent entities j $(1 \le j \le N-1)$ while a revised wholesale price contract is made for the two financing entities k and m. Pairwise revenue sharing contracts are discussed in [22] [23] [24]; however, they do not incorporate either a revised wholesale price contract or a multi-period financing setting, which makes this study unique.

Under RSRWPC, the revenue portion which is shared from supply chain entity j to $j+1$ $(1 \le j \le \mathrm{N}-2)$ is given by $\theta_{j\to j+1}$ where $|\theta_{j\to j+1}| < 1$. For the revised wholesale price contract, the loan rate, i, and supply chain entity k's retail price at time t, p_{kt} will be discounted by $r_{1-\mathrm{II}}$ and r_2, respectively. Discount rate values will be fixed in each period t.

To demonstrate the advantages of the proposed RSRWPC, there are two scenarios to be discussed and compared, seen in Figure 3-4. The first scenario is to apply the pairwise revenue sharing combined with only the loan rate being discounted (RSLRD); the second is to apply the pairwise revenue sharing but combined with revised wholesale price contract, where both loan rate and unit retail price are discounted (RSRWPC).

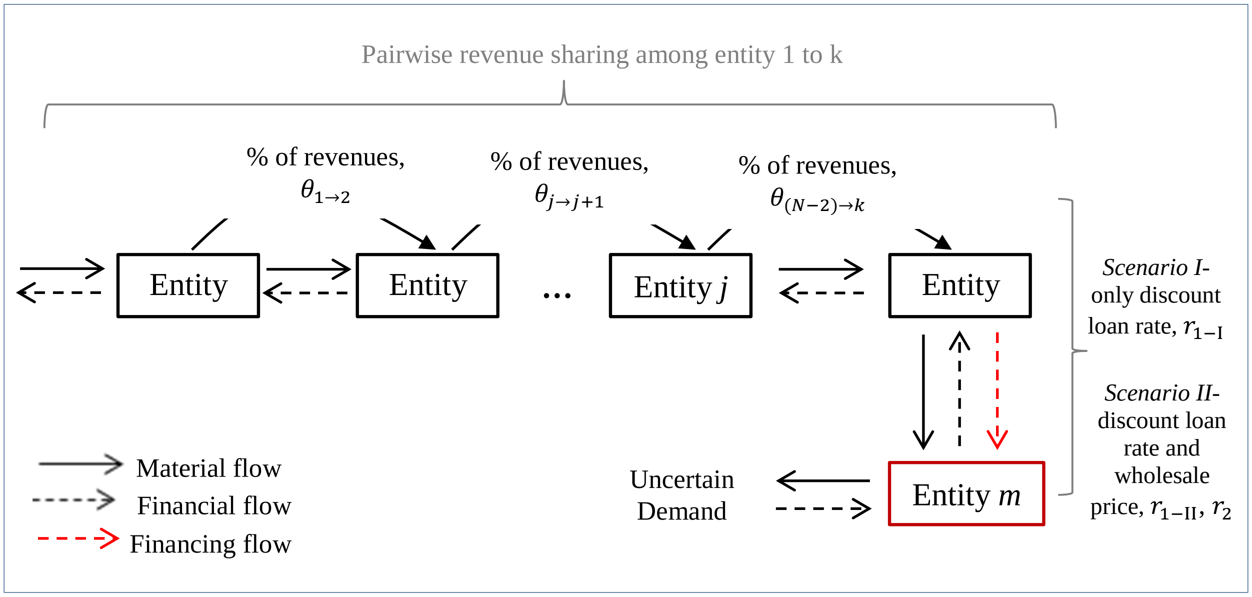

Figure 3-4: Coordination Scenarios I (RSLRD) and II (RSRWPC)

3.3.3.1.1 Scenario I –RSLRD

In this scenario, pairwise revenue sharing is employed between SC entities 1 through k in each period. The contract between each entity *j* and *j+1* is governed by the percentage of revenue to share, the discount rate of loan rate and the order quantity from entity *j*, *k*, and *m* which is given as $(\theta_{j\to j+1}, r_{1-\mathrm{I}}, Q_{jt}, Q_{kt}, Q_{mt})$. Since there is no wholesale price discount in this scenario, in order to guarantee the capital constrained entity $m's$ profit and efficiency, entity k need to lower the loan rate by use of the discount rate, $r_{1-\mathrm{I}}$ $(0 < r_{1-\mathrm{I}} < 1)$. The greater revenue portion received from upstream partner, the lower loan rate will be charged to downstream capital constrained entity j. The coordinated supply chain entity's average profit,

$\pi_{jT-I}, \pi_{kT-I}, \pi_{mT-I}$ as well as the overall supply chain profit, π_{SCT-I} in Scenario I are expressed in Equations 3.75 through 3.78.

$$\pi_{jT-I}(\theta_{j\to j+1}, Q_{jt}) = \frac{1}{T}\sum_{t=1}^{T}[(p_j - c_j)(1-\theta_{j\to j+1})Q_{(j+1)t} - t_j MIN(Q_{jt} + I_{j(t-1)}, Q_{(j+1)t}) - h_j(Q_{jt} + I_{j(t-1)} - Q_{(j+1)t})^+ - b_j(Q_{(j+1)t} - Q_{jt} - I_{j(t-1)})^+] \quad (3.75)$$

$$\pi_{kT-I}(\theta_{j\to j+1}, Q_{kt}, r_{1-\mathrm{I}}) = \frac{1}{T}\sum_{t=1}^{T}\Big[(p_k - c_k)Q_{(k+1)t} + (p_j - c_j)\theta_{j\to j+1}Q_{(j+1)t} + \sum_1^t R_t(A/P, i * r_{1-\mathrm{I}}, T-t+1) - t_k MIN(Q_{kt} + I_{k(t-1)}, Q_{(k+1)t}) - h_k(Q_{kt} + I_{k(t-1)} - Q_{(k+1)t})^+ - b_k(Q_{(k+1)t} - Q_{kt} - I_{k(t-1)})^+ - \sum_1^t R_t((A/P, \varepsilon, T-t+1) - (A/P, i * r_{1-\mathrm{I}}, T-t+1)) - R_t\Big] \quad (3.76)$$

$$\pi_{mT-I}(Q_{mt}, r_{1-\mathrm{I}}) = \frac{1}{T}\sum_{t=1}^{T}\Big[(p_m - c_m)D_t + R_t - \sum_1^t R_t(A/P, i * r_{1-\mathrm{I}}, T-t+1) - t_m MIN(Q_{mt} + I_{m(t-1)}, Q_{(m+1)t}) - h_m(Q_{mt} + I_{m(t-1)} - Q_{(m+1)t})^+ - b_m(Q_{(m+1)t} - Q_{mt} - I_{m(t-1)})^+\Big] \quad (3.77)$$

$$\pi_{scT-I}(\theta_{j\to j+1}, r_{1-\mathrm{I}}, Q_{jt}, Q_{kt}, Q_{mt}) = \pi_{jT-I}(\theta_{j\to j+1}, Q_{jt}) + \pi_{kT-I}(\theta_{j\to j+1}, Q_{kt}, r_{1-\mathrm{I}}) + \pi_{mT-I}(Q_{mt}, r_{1-\mathrm{I}}) \quad (3.78)$$

Based on the profit equations built above, the optimal average coordinated supply chain profit will be determined through the model shown in Equations 3.79 through 3.88. The constraints noted in Equations 3.84 through 3.86 ensure the coordinated supply chain entity efficiency is greater than non-coordinated efficiency. The decision variables are $\theta_{j\to j+1}$, Q_{jt}, Q_{kt}, Q_{mt}, and $r_{1-\mathbf{I}}$, noted in Table 3-3.

$$\text{MAX} \quad \pi_{scT-I}(\theta_{j\to j+1}, Q_{jt}, Q_{kt}, Q_{mt}, r_{1-\text{I}}) \tag{3.79}$$

$$\text{s.t. } MIN\left(Q_{jt} + I_{j(t-1)}, Q_{(j+1)t}\right) \geq s * Q_{(j+1)t}\,, \quad \forall j \in J, \forall t \in T \tag{3.80}$$

$$MIN\left(Q_{kt} + I_{k(t-1)}, Q_{(k+1)t}\right) \geq s * Q_{(k+1)t}\,, \quad \forall k \in M, \forall t \in T \tag{3.81}$$

$$MIN\left(Q_{mt} + I_{m(t-1)}, Q_{(m+1)t}\right) \geq s * Q_{(m+1)t}\,, \quad \forall m \in P, \forall t \in T \tag{3.82}$$

$$MAX\left(Q_{1t} + I_{1(t-1)}, Q_{2t}\right) \leq L + I_{1(t-1)}, \quad \forall t \in T \tag{3.83}$$

$$\eta_{jT-I} \geq \eta_{jT}, \quad \forall j \in J, \forall t \in T \tag{3.84}$$

$$\eta_{kT-I} \geq \eta_{kT}, \quad \forall k \in M, \forall t \in T \tag{3.85}$$

$$\eta_{mT-I} \geq \eta_{mT}, \quad \forall m \in P, \forall t \in T \tag{3.86}$$

$$T > 2 \tag{3.87}$$

$$0 < r_{1-\text{I}}, |\theta_{j\to j+1}| < 1 \tag{3.88}$$

3.3.3.1.2 Scenario II- RSRWPC

In this scenario, the contract is expanded by incorporating a discount rate for the wholesale price, r_2 for entity *k*, thus the contract becomes ($\theta_{j\to j+1}, Q_{jt}, Q_{kt}, Q_{mt}, r_{1-\text{II}}, \boldsymbol{r_2}$). Supply chain entity k will adjust both loan rate i, and sales price p_{kt}, to balance supply chain profit distribution. The coordinated supply chain entity's average profit, $\pi_{jT-II}, \pi_{kT-II}, \pi_{mT-II}$ as well as the overall coordinated supply chain profit, π_{SCT-II} in Scenario II are expressed in Equations 3.89 through 3.92.

$$\pi_{jT-II}(\theta_{j\to j+1}, Q_{jt}) = \frac{1}{T}\sum_{t=1}^{T}[\left(p_j - c_j\right)(1-\theta_{j\to j+1})Q_{(j+1)t} - t_j MIN\left(Q_{jt} + I_{j(t-1)}, Q_{(j+1)t}\right) - h_j\left(Q_{jt} + I_{j(t-1)} - Q_{(j+1)t}\right)^{+} - b_j(Q_{(j+1)t} - Q_{jt} - I_{j(t-1)})^{+}] \tag{3.89}$$

$$\pi_{kT-II}(\theta_{j\rightarrow j+1}, Q_{kt}, r_{1-\text{II}}, r_2) = \frac{1}{T}\sum_{t=1}^{T}\Big[(p_k r_2 - c_k)Q_{(k+1)t} + (p_j - c_j)\theta_{j\rightarrow j+1}Q_{(j+1)t} + \sum_1^t R_t(A/P, i * r_{1-\text{II}}, T-t+1) - t_k MIN(Q_{kt} + I_{k(t-1)}, Q_{(k+1)t}) - h_k(Q_{kt} + I_{k(t-1)} - Q_{(k+1)t})^+ - b_k(Q_{(k+1)t} - Q_{kt} - I_{k(t-1)})^+ - \sum_1^t R_t((A/P, \varepsilon, T-t+1) - (A/P, i * r_{1-\text{II}}, T-t+1)) - R_t\Big] \quad (3.90)$$

$$\pi_{mT-II}(Q_{mt}, r_{1-\text{II}}, r_2) = \frac{1}{T}\sum_{t=1}^{T}\Big[(p_m - c_m(p_k r_2))D_t + R_t - \sum_1^t R_t(A/P, i * r_{1-\text{II}}, T-t+1) - t_m MIN(Q_{mt} + I_{m(t-1)}, Q_{(m+1)t}) - h_m(Q_{mt} + I_{m(t-1)} - Q_{(m+1)t})^+ - b_m(Q_{(m+1)t} - Q_{mt} - I_{m(t-1)})^+\Big] \quad (3.91)$$

$$\pi_{scT-II}(\theta_{j\rightarrow j+1}, r_{1-II}, r_2, Q_{jt}, Q_{kt}, Q_{mt}) = \pi_{jT-II}(\theta_{j\rightarrow j+1}, Q_{jt}) + \pi_{kT-II}(\theta_{j\rightarrow j+1}, Q_{kt}, r_{1-II}, r_2) + \pi_{mT-II}(Q_{mt}, r_{1-II}, r_2) \quad (3.92)$$

Similar to Scenario I, the mathematical model to optimize overall average supply chain profit in Scenario II is shown in Equations 3.93 through 3.102. Constraints 3.98 through 3.100 ensure the coordinated scenario yields greater efficiency than the non-coordinated scheme. The decision variables are $\theta_{j\rightarrow j+1}$, Q_{jt}, Q_{kt}, Q_{mt}, $r_{1-\text{II}}$, and r_2 noted in Table 3-3.

$$\text{MAX} \quad \pi_{scT-II}(\theta_{j\rightarrow j+1}, Q_{jt}, Q_{kt}, Q_{mt}, r_{1-\text{I}}, r_2) \quad (3.93)$$

$$\text{s.t. } MIN\ (Q_{jt} + I_{j(t-1)}, Q_{(j+1)t}) \geq s * Q_{(j+1)t}\ , \qquad \forall j \in J, \forall t \in T \quad (3.94)$$

$$MIN\ (Q_{kt} + I_{k(t-1)}, Q_{(k+1)t}) \geq s * Q_{(k+1)t}\ , \qquad \forall k \in M, \forall t \in T \quad (3.95)$$

$$MIN\ (Q_{mt} + I_{m(t-1)}, Q_{(m+1)t}) \geq s * Q_{(m+1)t}\ , \qquad \forall m \in P, \forall t \in T \quad (3.96)$$

$$MAX\ (Q_{1t} + I_{1(t-1)}, Q_{2t}) \leq L + I_{1(t-1)}, \qquad \forall t \in T \quad (3.97)$$

$$\eta_{jT-II} \geq \eta_{jT}, \qquad \forall j \in J, \forall t \in T \quad (3.98)$$

$$\eta_{kT-II} \geq \eta_{kT}, \qquad \forall k \in M, \forall t \in T \qquad (3.99)$$

$$\eta_{mT-II} \geq \eta_{mT}, \qquad \forall m \in P, \forall t \in T \qquad (3.100)$$

$$T > 2 \qquad (3.101)$$

$$0 < r_{1-\mathrm{II}}, r_2, |\theta_{j \to j+1}| < 1 \qquad (3.102)$$

As discussed by Lariviere (1999) and Petruzzi and Dada (1999), an analytical solution for maximizing supply chain profit with multiple entities is intractable. Therefore, a numerical example will be analyzed in Chapter 4.3, using the methodology discussed above.

Chapter 4: Results and Analysis

To verify the general conclusions developed in Chapter 3, three numerical examples are presented in this chapter. The first example explores the optimal financing option and supply chain profitability under various demand risk levels for a two-echelon supply chain. The second example explores the effective coordination contract to improve overall supply chain performance and satisfy different supply chain entities in the two-echelon supply chain. The third example demonstrates the multi-echelon, multi-period supply chain financing scenario and compares the profits and efficiencies with and without financing coordination.

4.1 Numerical Example in Two-Echelon Supply Chain Financing

To demonstrate the effect of financing and risk on the profit of the supply chain members, simulations are performed in @Risk. The supply chain consists of a capital-constrained retailer who cannot afford to purchase the optimal order quantity. Additionally, it is assumed that due to the retailer's credit rating, the principal needed to bring the order quantity to optimal may or may not be feasible. The retailer faces three choices: borrow no money and purchase a sub-optimal amount, borrow money from the supplier (at either the optimal or a sub-optimal principal) or borrow money from a third party financial institution (at either the optimal or a sub-optimal principal). The optimal principal will bring the order quantity to the optimal value, while a sub-optimal principal (due to the retailer's credit rating) will necessitate a sub-optimal order quantity.

4.1.1 Simulation Parameter Development

Based on the models built (Equations 3.1 - 3.23), the software @Risk is used to simulate 10,000 instances of each scenario with the parameters described in Table 4-1 below.

Table 4-1: Parameters: Two-Echelon SCF under Stochastic Demand Risk

Parameter	Value
s_r	\$1200/unit
c_r	\$610/unit
c_s	\$300/unit
s_s	\$600/unit
v_r	\$550/unit
g_s	\$350/unit
g_r	\$640/unit
t	1 year
i_β	7%/year
i_γ	U (4%/year, 6%/year)
i_ε	N (i_β,σ_2)
M_∂	\$1,800,000
μ_1	5000 units

One of the main objectives of this paper is to study the effects of risk on supply chain financing. Risk is defined by the coefficient of variation of demand, c_v and used to determine the standard deviation, by holding the demand mean constant. Here, the coefficient of variation is studied in the range of: c_v = *[0.10, 0.25]* in increments of 0.01, giving 16 demand risk levels.

The interest rate charged by the third party institution (i_β =7%/year) is taken as an approximate average of small business loan rates over the last few years [97]. The supplier's interest rate, i_γ has a maximum of i_β since it is assumed the supplier has an incentive to retain the retailer by offering capital. Supplier's market investment rate, i_{ε} , which is the interest the supplier will gain by investing capital in the market, rather than tying it up with the retailer, follows a normal distribution N (i_{γ},σ_2). The coefficient of variation is used for the volatile investment rate, so that, as in demand, the mean is constant and the standard deviation is determined by the risk level.

4.1.2 Simulation Results and Managerial Implications

Simulations are performed using @Risk in Excel. The results of the 10,000 simulations yielded some interesting results for various risk levels. The supply chain profit, retailer's profit and supplier's profit are shown in three cases, respectively. For reference, Case I is no financing, Case II is third-party financing and Case III is supplier-led financing.

$3,000,000 is used as the value for R_{β} and R_{γ} when optimal financing occurs, while $1,500,000 for R_{β} and R_{γ} when sub-optimal financing occurs from either third party financial institution or the supplier in this numerical simulation (Note: R refers to retailer, S refers to supplier, while SC refer to supply chain).

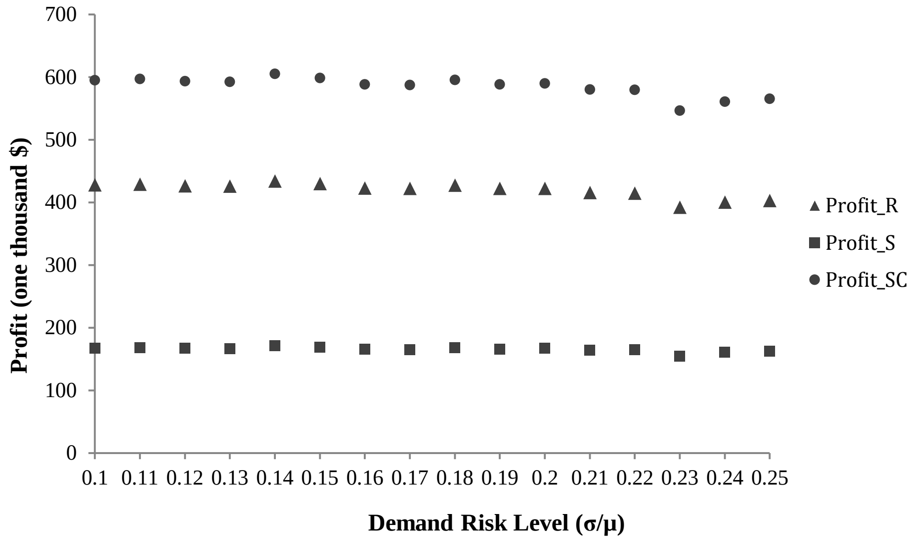

Figure 4-1: Profits of Supply Chain, Retailer and Supplier in Case I (No Financing)

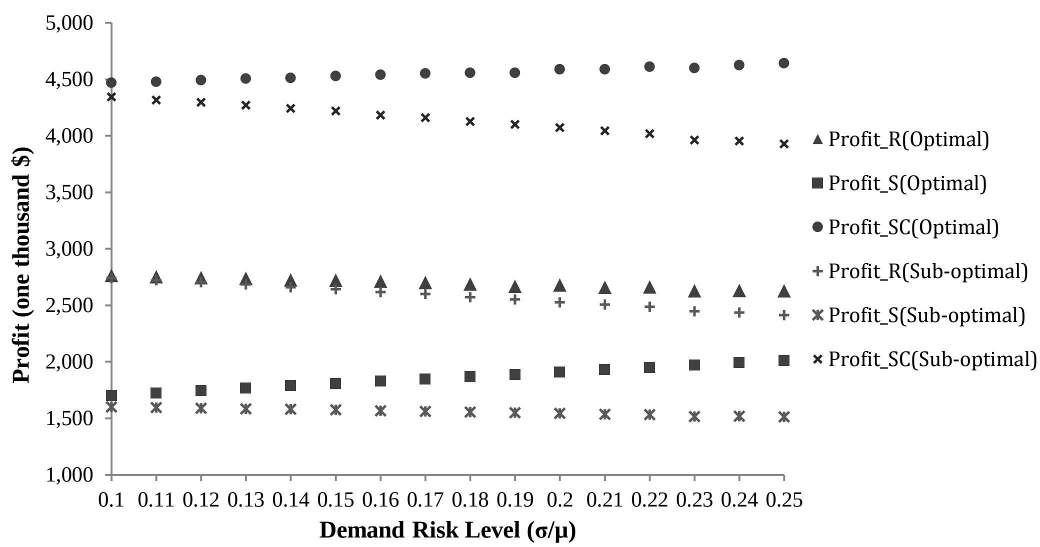

Figure 4-2: Profits of Supply Chain, Retailer and Supplier in Case II (Third Party Financing)

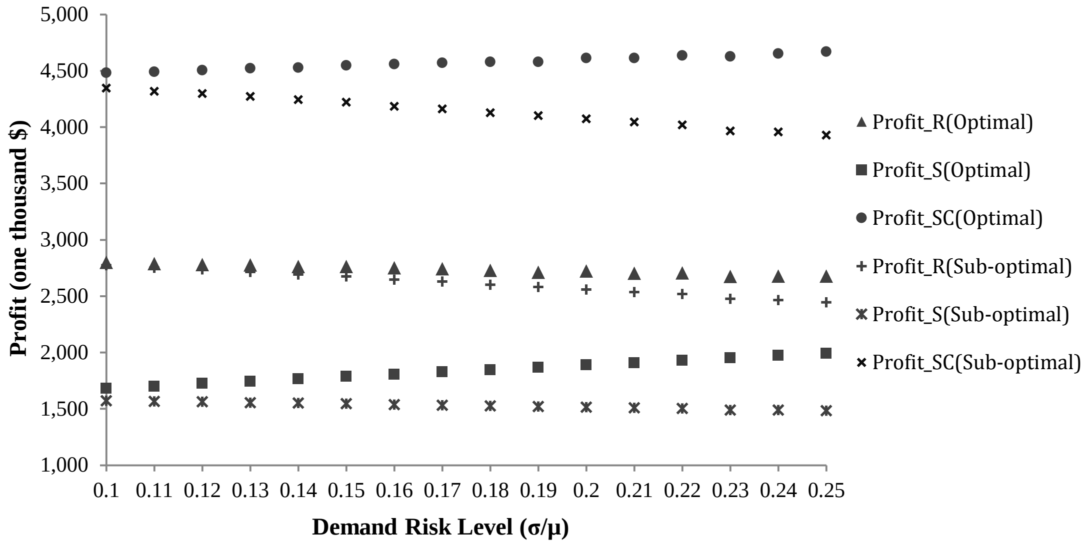

Figure 4-3: Profits of Supply Chain, Retailer and Supplier in Case III (Supplier Financing)

Figures 4-1 to 4-3 demonstrate how the profits of retailer, supplier and supply chain vary with the demand risk levels in each case, and verify Propositions 2, 4, and 6, respectively. Figures 4-4 to 4-6 are plotted to identify in what cases the supply chain, retailer or supplier can achieve the maximum profit.

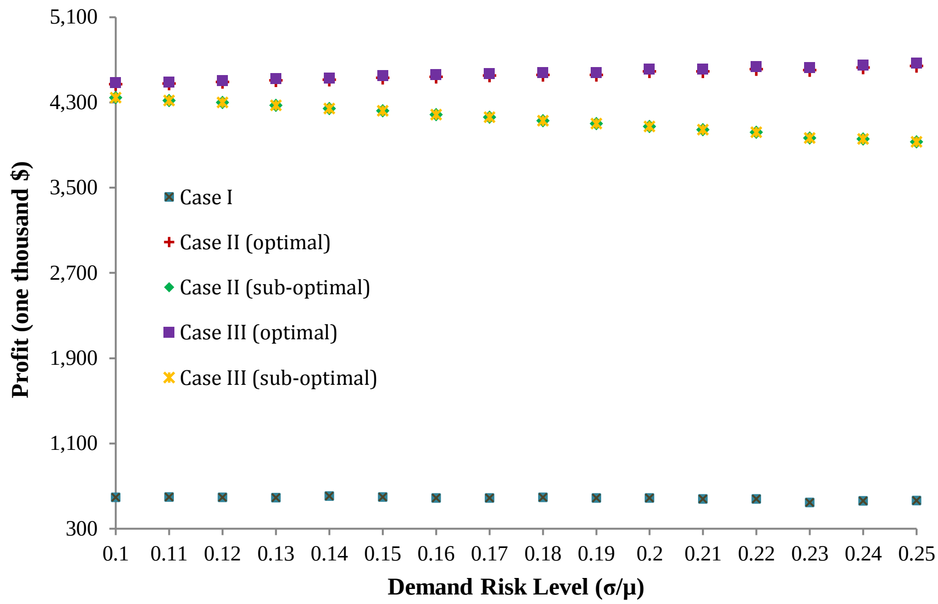

Figure 4-4: Profit of Supply Chain in Each Case

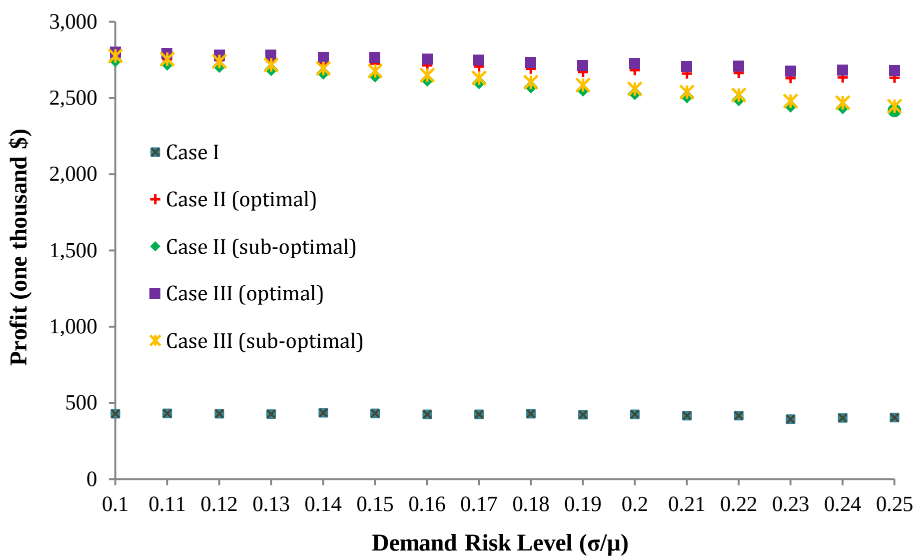

Figure 4-5: Profit of Retailer in Each Case

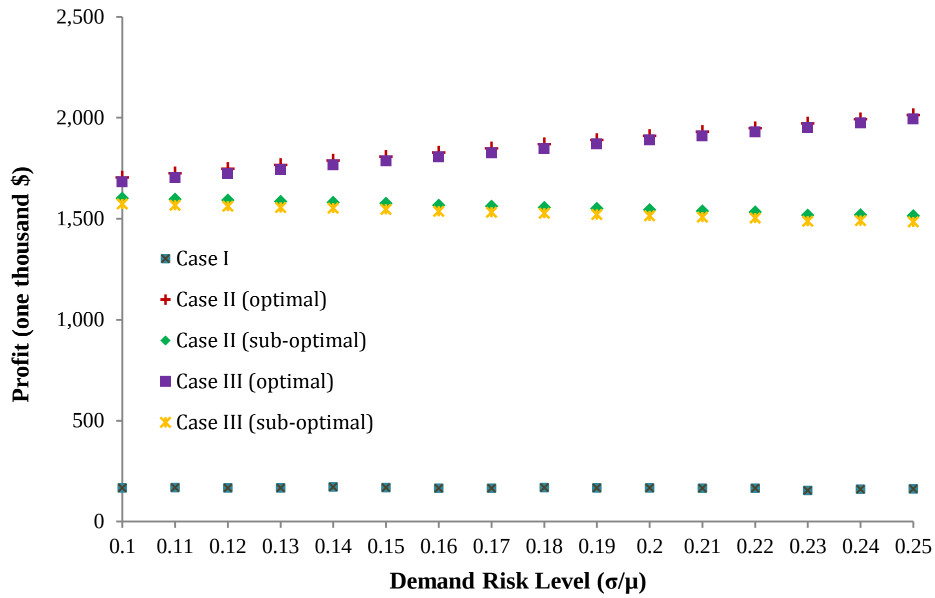

Figure 4-6: Profit of Supplier in Each Case

From the numerical simulation, a few important managerial findings are noticed:

- Profits of the retailer, supplier, and supply chain are minimized with no financing, while supply chain profit is maximized with supplier financing assuming optimal capital can be borrowed.
- Compared with the supplier and supply chain, the retailer profit is more robust to risk.
- The borrowed amount of capital affects the profits of all the three parities (retailer, supplier and supply chain). Optimal amount of capital can result in better profitability than sub-optimal condition.

These results have some interesting managerial implications. First of all, any supply chain, aiming to optimize the profit, should encourage the capital-constrained retailer to finance to increase order quantity for the best of all supply chain members. Which financing channel to choose is a function of many factors, such as loan rate, available amount of capital, and macro market development. However, under equal conditions, financing from supplier can result in better supply chain performance than financing from third party financial institutions. To balance the profit inside the chain, a game theory approach is needed to generate a contract between chain members to satisfy their individual profit requirement while improve the whole supply chain performance.

4.2 Numerical Study in Two-Echelon Supply Chain Coordination

A numerical example is illustrated through applying Modified Wholesale Price Contract (MWPC) to coordinate two-echelon supply chain internal financing, where a capital constrained retailer needs to finance from upstream supplier to bring order quantity to be optimal. The profits and efficiencies are developed and compared in different scenarios, where conclusions are generated.

4.2.1 Parameter Development

Model parameter values are described in Table 4-2 below. Python is used to explore the function solutions for Equations 3.24 through 3.59.

Table 4-2: Stackelberg Game Parameters: Two-Echelon SCF

Parameter	Value
p_R	$30/unit
p_S	$20/unit
D	N(100,10)
c_S	$10/unit
c_R	$20/unit
g_R	$12/unit
g_S	$12/unit
v_R	$8/unit
i_ϵ	7%/year
i_R	6.5%/year
R_R	$1800
R_S	$1000
t	1 year

4.2.2 Two-Echelon Supply Chain Coordination Results Analysis

Stackelberg results in this numerical example are explored and compared in Traditional Newsvendor Model with Financing (TNMF), coordinated supply chain with Modified Wholesale Price Contract (MWPC), coordinated supply chain with Traditional Wholesale Price Contract (TWPC), respectively.

4.2.2.1 Stackelberg Results under TNMF

Both retailer-led and supplier-led Stackelberg results in TNMF are shown in Table 4-3, which are then utilized to compare with the profits and efficiencies of the retailer, supplier and supply chain in MWPC. The retailer yields greater profit and efficiency in RS-TNMF while the supplier suffers from lower profit and efficiency in SS-TWPC. In general, overall supply chain profit and efficiency are greater in the RS-TNMF than the SS-TNMF.

Table 4-3: Retailer-led and Supplier-led Stackelberg Results under TNMF

	Q^*	${p_S}^*$	i_R^*	η_{SC}^*	π_R^*	π_S^*	π_{SC}^*
Retailer-led	113	\$15.93	R	94.59%	\$1330.79	\$639.16	\$1969.95
Supplier-led	103	\$20.00	6.50%	91.47%	\$856.40	\$1048.49	\$1904.89

Note: *R=Real*

The change in supply chain entities' profits and efficiencies are analyzed with respect to the wholesale price, p_S. The results are shown in Figures 4-7 and 4-8, verifying Corollary 11. Though the profit and efficiency of the retailer decrease in the wholesale price in a long run, there is no consistent trend. Similar to the retailer, the supplier's profit and efficiency increase in the wholesale price but with no consistent trend. In a long run, the increasing wholesale price will increase the retailer's order costs while increase the supplier's sales revenue, which explains the changing trends of the two entities. For the supply chain profit and efficiency, as proved in Corollary 11, can be maximized by the wholesale price.

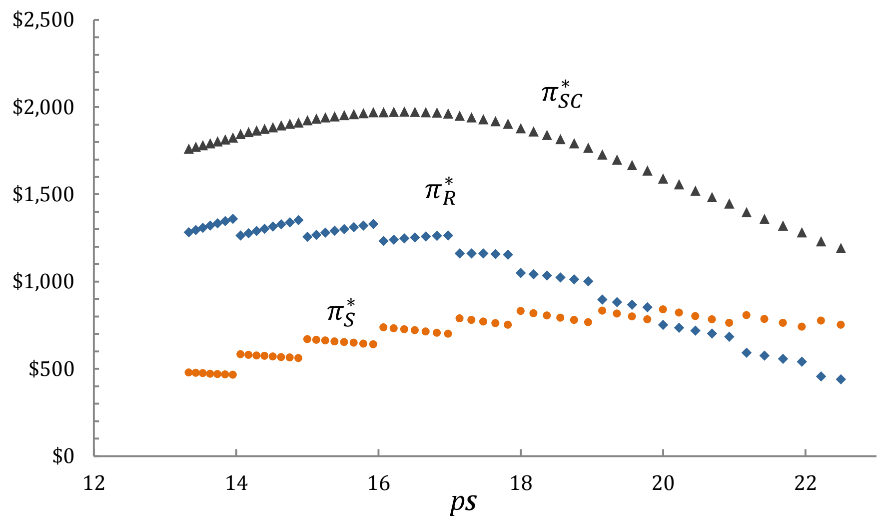

Figure 4-7: Effect of Wholesale Price, $\boldsymbol{p_S}$, on Profits under RS-TNMF

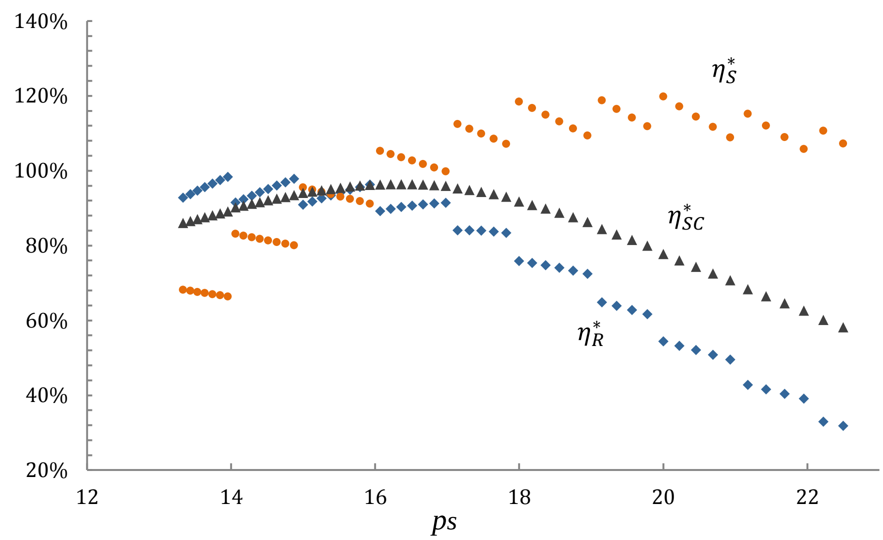

Figure 4-8: Effect of Wholesale Price, $\boldsymbol{p_S}$, on Efficiency under RS-TNMF

Figures 4-9 and 4-10 show the dependence of the incremental profits and efficiencies (retailer, supplier and supply chain) on the order quantity, Q, under SS-TWPC. The retailer and supply chain profits are maximized under order quantity, Q, with the optimal SC order quantity smaller than the optimal retailer order quantity. Supplier's profit is monotonically increasing in Q, verifying Corollary 12.

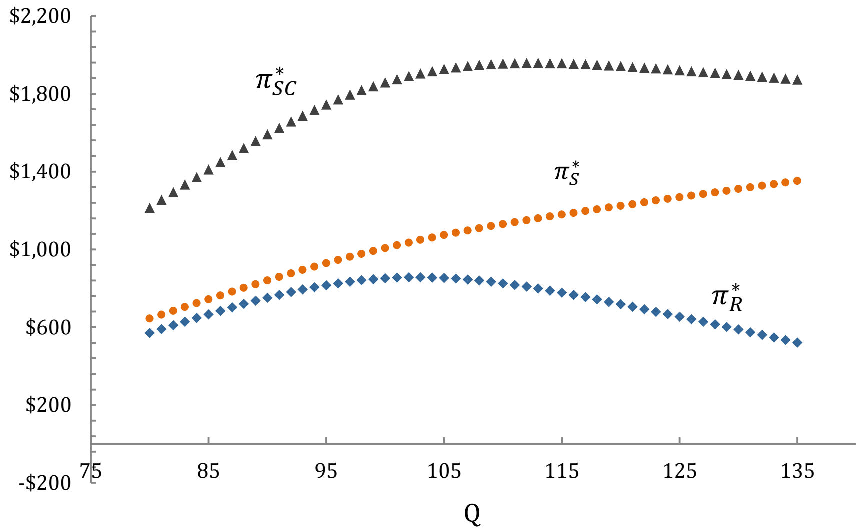

Figure 4-9: Effect of Order Quantity, *Q*, on Profit under SS-TNMF

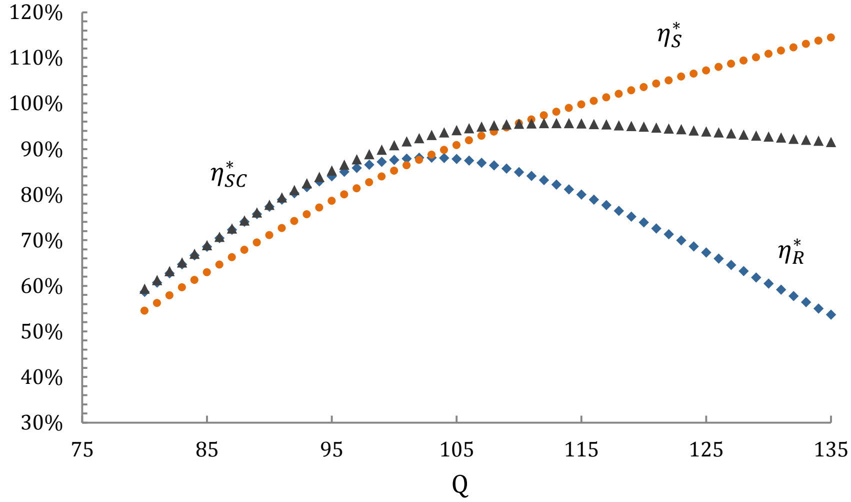

Figure 4-10: Effect of Order Quantity, *Q*, on Efficiency under SS-TNMF

4.2.2.2 Stackelberg Results under MWPC

Both the retailer-led and supplier-led Stackelberg results under MWPC are shown in Table 4-4. Compared with the Stackelberg results under TWPC (see Table 4-5), the profits and efficiencies of the retailer, supplier, and supply chain are all increased and in particular it is noted that the supply chain efficiencies achieve as high as 98.30% in both supplier-led and retailer-led Stackelberg games, though the transfer payment is different. The supplier requires a higher transfer payment in SS-MWPC to guarantee an increased profit and efficiency. Further, the retailer order quantity is increased in both the RS-and SS-MWPC which improves overall supply chain performance.

It is noted that, the supply chain coordination performance under both MWPC and TWPC, shown in Table 4-4 and 4-5 are greater than that under TNMF, shown in Table 4-3.

Table 4-4: Retailer-led and Supplier-led Stackelberg Results under MWPC

	Q^*	w^*	i^*	λ	T	η_{SC}^*	π_R^*	π_S^*	π_{SC}^*
Retailer-led	109	$2.0	0.65%	0.11	$1250	98.30%	$1381.98	$665.22	$2047.20
Supplier-led					$1660	98.30%	$971.98	$1075.22	$2047.20

Note: *"T" means transfer amount from retailer to supplier as reimbursements of the low wholesale price and rate*

The Stackelberg game using TWPC only applies a discount to the wholesale price without loan rate discount and transfer payment included. While the overall supply chain profits and efficiencies are increased over TNMF. They are outperformed by the MWPC. The modified contract yields 98.30% efficiencies while the traditional one has the efficiencies of 94.80% and 92.59% in supplier-led and retailer-led Stackelberg, respectively. It is also noticed that, resources are more evenly distributed between the supplier and retailer with the modified contract than the traditional contract.

Table 4-5: Retailer-led and Supplier-led Stackelberg Results under TWPC

	Q^*	w^*	i^*	λ	η_{SC}^*	π_R^*	π_S^*	π_{SC}^*
Retailer-led	106	$14.6	6.50%	0.73	94.80%	$1408.84	$565.40	$1974.24
Supplier-led	104	$18.6	6.50%	0.93	92.59%	$998.46	$929.88	$1928.35

Figures 4-11 and 4-12 show the intervals for the contract parameter, λ, during which both the supplier and retailer are better off by utilizing the proposed MWPC. It is clearly illustrated that retailer's profit increases with the discount rate while the supplier's profit decreases with it in both RS-MWPC and SS-MWPC. Under retailer-led Stackelberg, $0.09 \leq \lambda \leq 0.13$ can bring both the retailer and supplier increased profits. This is similar to the supplier-led Stackelberg with the interval of $0.09 \leq \lambda \leq 0.16$ that makes increased profits. Figures 4-11 and 4-12 verify Corollary 13. To better illustrate the plots below:

$\Delta\pi S^* = (\pi^*_{S-contract} - \pi^*_{S-noncontract})/\pi^*_{S-noncontract}$,

$\Delta\pi R^* = (\pi^*_{R-contract} - \pi^*_{R-noncontract})/ \pi^*_{R-noncontract}$

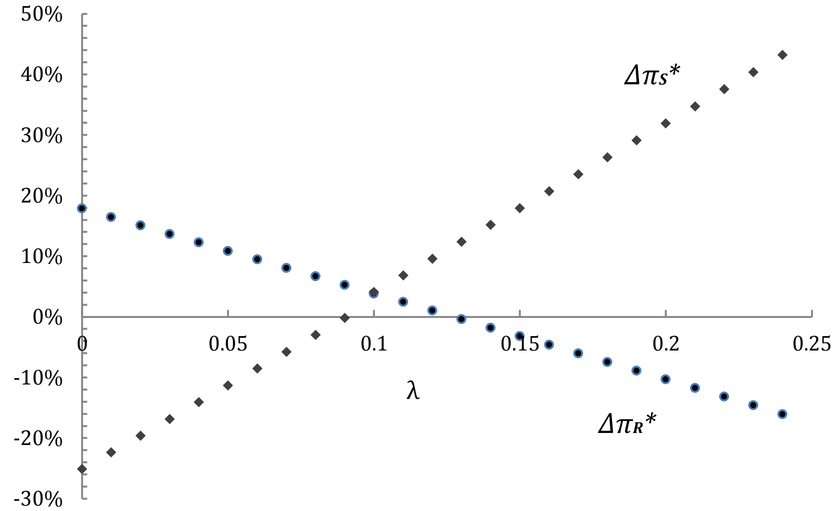

Figure 4-11: Profit Difference between Retailer-led Stackelberg With and Without Contract, as a Function of Discount %, λ

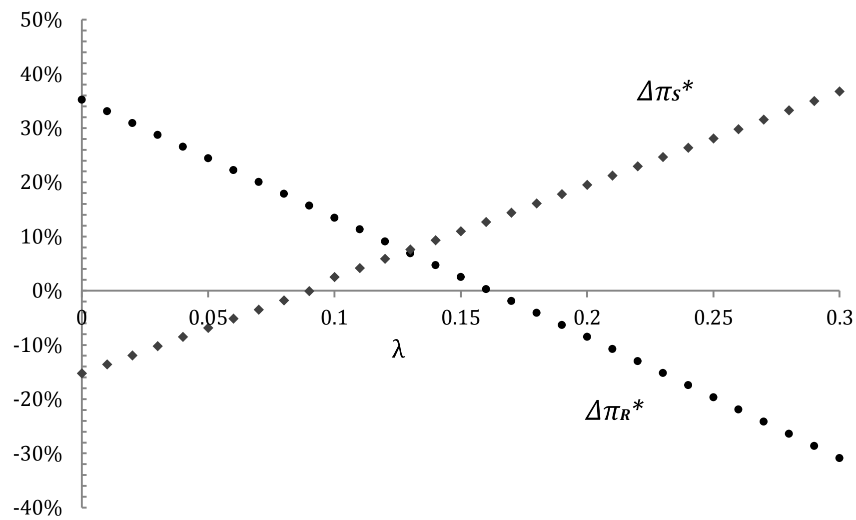

Figure 4-12: Profit Difference between Supplier-led Stackelberg With and Without Contract, as a Function of Discount %, λ

4.2.3 Managerial Implications from Coordinating Two-Echelon SCF

Optimizing the supply chain in general does not guarantee optimality for the individual members, and quite often causes large disparities from optimality for some members. The proposed modified wholesale price contract does lessen this efficiency disparity among supply chain members and increases overall supply chain profit and efficiency, which can be generally applied to budget-intense supply chain related situations to add value for the chain. The very important issue related with implementing coordination contract lies in the proper interval for contract parameters. Inappropriate parameter value can lead to unsatisfying results. The research shows the process to explore contract parameter using Stackelberg strategy, which is a generalized thought and could be widely applied in practice. It is worthy noting that, Stackelberg leader who has great negotiation power does not always yield greater profit and efficiency compared with as a follower, which instructs the supply chain entities not to spend that much time and energy fighting for being a negotiation leader but to work on the optimal coordination mechanism to benefit the whole chain.

4.3 Numerical Study on Multi-Echelon, Multi-Period Supply Chain Financing

This numerical example studies a three-echelon supply chain ($N = 3$) seen in Figure 4-13, consisting of raw material supplier, manufacturer, and retailer, where the retailer is capital constrained and the liquid manufacturer provides financial support to the retailer at a specified loan rate. T study periods ($3 \leq T \leq 10$) are chosen to explore the financial strategies and coordination effects. In each period, the retailer determines the optimal order quantity, and based upon the previous period's profits (determined from stochastic demand), the amount to be financed is calculated. In Scenario I, only the revenue sharing portion and loan rate discount are determined, while in Scenario II, the revenue sharing portion, loan rate discount and wholesale price discount are determined. It is assumed that the loan will be an installment payment loan and all principal and interest should be paid by the end of contract period. Simulations and optimization are applied to analyze profits and efficiencies of supply chain entities as well as the overall supply chain, where interesting findings and managerial insights are generated.

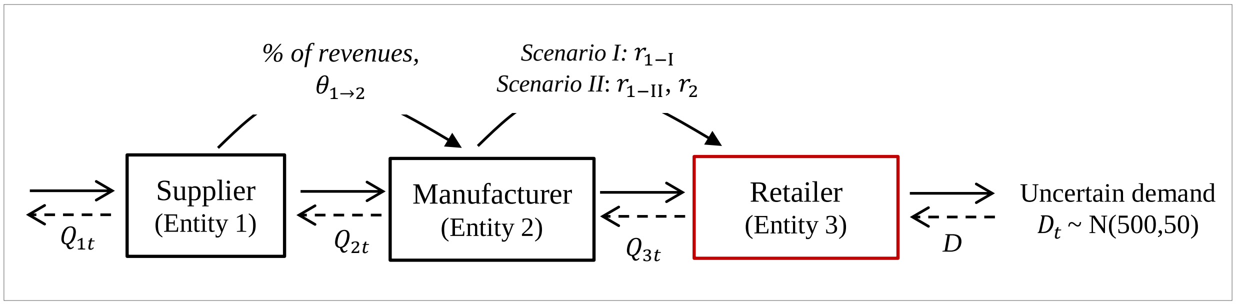

Figure 4-13: Thee-echelon, T-period Supply Chain Model ($3 \leq T \leq 10$)

4.3.1 Experimental Parameters

The model parameters are shown in Table 4-6 below. Extended LINGO 15.0 (unrestricted version) was used to perform the optimization and 5000 simulations were performed for each scenario. The results given are the average values.

Table 4-6: Parameters: Multi-Echelon, Multi-Period SCF

Parameter	Value
s	98%
c_1	\$15/unit
p_1	\$30/unit
t_1	\$0.5/unit
h_1	\$0.1/unit
b_1	\$2.0/unit
L	2000 units
c_2	\$35/unit
p_2	\$80/unit
t_2	\$0.6/unit
h_2	\$0.2/unit
b_2	\$1.8/unit
c_3	\$80/unit
p_3	\$160/unit
t_3	\$0.8/unit
h_3	\$0.3/unit
b_3	\$1.5/unit
ε	5%/ period

4.3.2 Multi-Echelon, Multi-Period Optimization with Financing (No Coordination)

In order to determine the optimal contract period for the supply chain, Equations 3.60 through 3.70 were utilized assuming that market demand follows an independent and identical normal distribution. The optimal average supply chain profit in contract period T, π_{scT-c} have 5000 results which are plotted in Figure 4-14 for study periods $T = 3\ to\ 10$. The value of the mean and mode are shown in each plot corresponding with period, *T*. The optimal average supply chain profit, π_{scT-c} is maximized in period 7, therefore $T_{sc}^{*} = 7$.

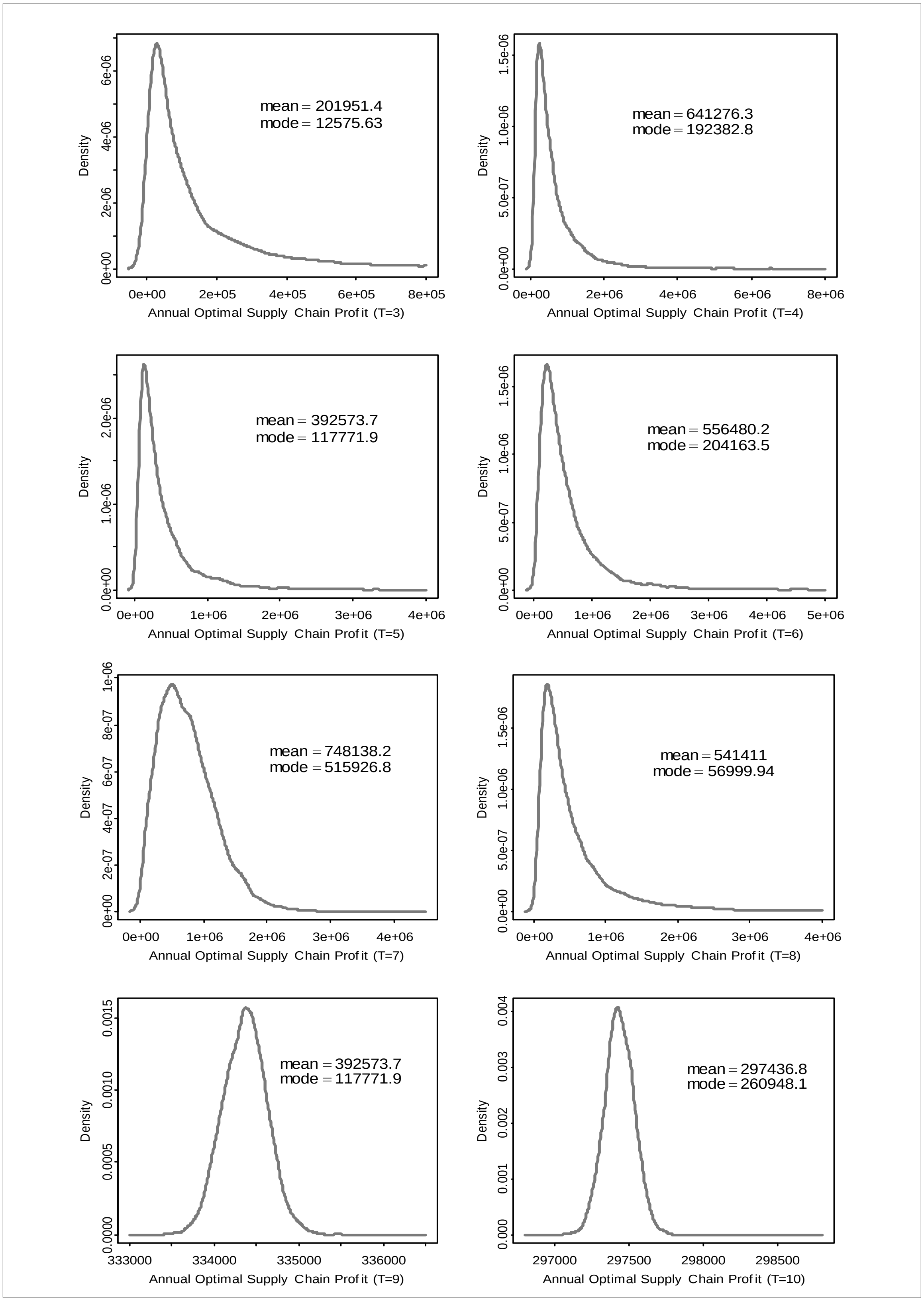

Figure 4-14: Optimal Average Supply Chain Profits in Periods $T = 3$ to 10

The efficiencies of the supplier, manufacturer, and retailer are also calculated for T = 3 to 10 periods. Table 4-7 demonstrates the mean values of the optimal average profits and actual average profits for three supply chain entities from T = 3 to 10 periods respectively. Table 4-8 shows the optimal average supply chain profit (mean value of the 5000 simulations) and the efficiencies of each supply chain entity, where the optimal value is highlighted in bold. π_{scT-c} and η_{1T}, η_{2T}, η_{3T} are also plotted in Figure 4-15.

Table 4-7: Supply Chain Entity Optimal and Actual Average Profits

T	π_{1T}^*	π_{2T}^*	π_{3T}^*	π_{1T-c}	π_{2T-c}	π_{3T-c}	i_{-c}
3	$193,810.43	$52,167.55	$36,658.59	$154,979.48	$45,376.67	$27,794.00	10.68%
4	$253,724.33	$42,666.94	$32,918.10	$232,386.41	$34,177.20	$28,818.24	16.21%
5	$369,318.67	$46,405.66	$31,456.99	$333,575.17	$32,086.32	$27,413.90	19.42%
6	$424,236.45	$46,288.17	$30,400.50	$336,756.50	$31,366.35	$25,587.50	20.54%
7	$707,331.85	$41,212.00	$29,905.88	$686,986.00	$25,249.42	$25,165.52	21.36%
8	$479,278.26	$46,780.40	$29,670.02	$417,280.49	$31,858.53	$25,818.99	22.18%
9	$318,961.00	$30,024.58	$28,308.61	$293,411.10	$17,595.59	$14,904.14	23.09%
10	$247,983.91	$42,706.81	$28,190.73	$239,931.23	$32,930.30	$24,575.28	23.42%

Table 4-8: Average Supply Chain Optimization Results for $T = 3$ to 10

T	π_{scT-c}	η_{1T-c}	η_{2T-c}	η_{3T-c}
3	$201,951.40	79.96%	86.98%	75.82%
4	$298,534.72	91.59%	80.10%	87.55%
5	$392,573.00	90.32%	69.14%	87.15%
6	$556,480.20	79.38%	67.76%	84.17%
7	$732,729.30	97.12%	61.27%	84.15%
8	$541,411.02	87.06%	68.10%	87.02%
9	$334,365.95	91.99%	58.60%	52.65%
10	$297,436.81	96.75%	77.11%	87.18%

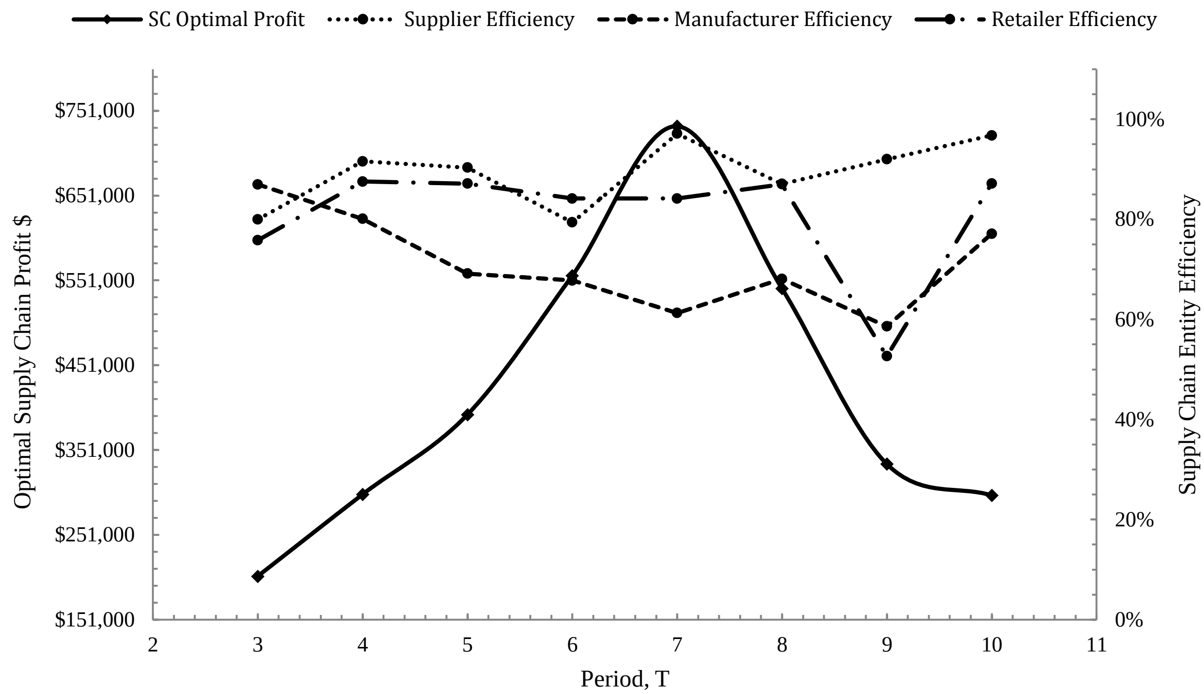

Figure 4-15: Optimal Average SC Profit and Actual Entity Efficiencies over T Periods

Figures 4-15 shows that π_{scT-c} is maximized at $T = 7$, which means the optimal number of contract periods for this specific capital constrained supply chain is seven. However, three supply chain entities may not all agree with this value when maximizing their own individual efficiency. It is shown that while the supplier does agree with $T = 7$ as the contracted business period time, the manufacturer and retailer prefer 3 and 4 periods, respectively. To diminish the negative disagreements, supply chain coordination mechanism is needed to coordinate supply chain actions under $T_{sc}^{*} = 7$.

According to Equation 3.74 analyzed in Section 3.3.3, the non-cooperative supply chain performance under $T = 7$ is shown in Table 4-9. The order quantities for the three supply chain entities in period $t, Q_{1t}, Q_{2t}, Q_{3t}$ are listed in Appendixes 6.

Table 4-9: Non-Cooperative Supply Chain Performance under $T = 7$

π_{scT-d}	η_{scT-d}	η_{1-d}	$\eta_{2-\mathrm{d}}$	$\eta_{3-\mathrm{d}}$	$\pi_{1-\mathrm{d}}$	$\pi_{2-\mathrm{d}}$	π_{3-d}	i_{-d}
$663,778.94	90.59%	86.36 %	64.44 %	88.09 %	$610,877.23	$26,558.12	$26,343.59	18.16%

4.3.3 Multi-Echelon, Multi-Period Optimization with Financing and Coordination

Since this research is interested in determining the effect of the revenue sharing, $\theta_{1\to2}$, interest rate, i, and wholesale price p_2 on supply chain profits, the remaining parameters will be fixed in this study. While order fill rate, s, maximum number of raw materials, L in each period, market investment rate, ε, transportation cost, t, holding cost, h , and backorder cost, b, will influence profit, it is not the intent of this study. Additionally, the particular profit split chosen will also depend on qualitative factors such as the firms' relative bargaining power. As the manufacturer's bargaining position becomes stronger, $\theta_{1\to2}$ would increase. In this study, all entities are considered equal.

The coordination effects resulting from optimizing Scenario I ($\theta_{1\to2}, r_{1-\mathrm{I}}, Q_{1t}, Q_{2t}, Q_{3t}$), Equations 3.75 through 3.88, and II ($\theta_{1\to2}, r_{1-\mathrm{II}}, r_2, Q_{1t}, Q_{2t}, Q_{3t}$), Equations 3.89 through 3.102 are demonstrated in Table 4-10 and Table 4-11, respectively. The efficiencies, $\eta_{x-I/II}$, and profits $\pi_{x-I/II}$ (where $x =$ SC entity and $I/II =$coordination in Scenario I and II) are shown in Table 4-10 and 4-11. In Scenario I, there are a total of five feasible solutions, among which the efficiency and profit are maximized when $\theta_{1\to2} = 0.18, r_1 = 42.61\%$. There are ten feasible coordination solutions in Scenario II, with the optimal solution when $\theta_{1\to2} = 0.19, r_1 = 25.48\%, r_2 = 90.64\%$. The order quantities for the three supply chain entities in period $t, Q_{1t}, Q_{2t}, Q_{3t}$ for Scenario I and II are in Appendixes 7 and 8, respectively.

Table 4-10: Profits and Efficiencies: Coordinating the RSLRD (Scenario I)

$\theta_{1\to 2}$	$r_{1-\mathrm{I}}$	$\pi_{sc-\mathrm{I}}$	$\eta_{1-\mathrm{I}}$	$\eta_{2-\mathrm{I}}$	$\eta_{3-\mathrm{I}}$	$\pi_{1-\mathrm{I}}$	$\pi_{2-\mathrm{I}}$	$\pi_{3-\mathrm{I}}$
0.14	48.34%	$688,892.78	89.09%	79.24%	87.14%	$630,175.85	$32,657.28	$26,059.65
0.18	*42.61%*	*$700,344.06*	*89.91%*	*90.46%*	*90.67%*	*$635,947.61*	*$37,279.69*	*$27,116.76*
0.31	42.28%	$687,947.48	88.51%	85.92%	88.44%	$626,088.27	$35,409.54	$26,449.67
0.32	41.13%	$681,892.69	87.73%	84.57%	88.66%	$620,524.26	$34,854.29	$26,514.14
0.33	38.07%	$677,882.68	87.37%	80.45%	89.42%	$617,985.36	$33,156.32	$26,741.00

Table 4-11: Profits and Efficiencies: Coordinating the RSRWPC (Scenario II)

$\theta_{1\to 2}$	$r_{1-\mathrm{II}}$	r_2	$\pi_{sc-\mathrm{II}}$	$\eta_{1-\mathrm{II}}$	$\eta_{2-\mathrm{II}}$	$\eta_{3-\mathrm{II}}$	$\pi_{1-\mathrm{II}}$	$\pi_{2-\mathrm{II}}$	$\pi_{3-\mathrm{II}}$
0.06	35.31 %	97.51 %	$712,964.16	92.23 %	80.05 %	92.33 %	$652,360.15	$32,990.51	$27,613.50
0.12	21.26 %	94.94 %	$710,387.92	91.60 %	86.33 %	90.04 %	$647,883.45	$35,576.59	$26,927.88
0.14	26.71 %	93.32 %	$711,827.77	91.67 %	89.38 %	88.86 %	$648,419.26	$36,835.64	$26,572.87
0.15	22.64 %	91.26 %	$711,058.38	91.46 %	87.94 %	93.26 %	$646,924.67	$36,242.68	$27,891.03
0.16	32.55 %	92.77 %	$709,653.77	91.26 %	89.51 %	91.02 %	$645,545.39	$36,889.47	$27,218.91
0.18	30.14 %	90.44 %	$702,844.06	90.52 %	87.30 %	89.00 %	$640,247.61	$35,979.69	$26,616.76
0.19	*25.48 %*	*90.64 %*	*$719,596.84*	*92.31 %*	*93.87 %*	*93.56 %*	*$652,930.62*	*$38,686.44*	*$27,979.78*
0.28	19.42 %	88.55 %	$693,755.86	89.43 %	87.60 %	83.98 %	$632,541.03	$36,100.87	$25,113.96
0.31	31.32 %	86.12 %	$687,177.48	88.75 %	83.49 %	83.53 %	$627,788.27	$34,409.54	$24,979.67
0.32	27.76 %	85.38 %	$680,892.69	87.87 %	84.57 %	81.97 %	$621,524.26	$34,854.29	$24,514.14

The coordination effects resulting from RSLRD, RSRWPC as well as the results without coordination are shown in Table 4-12 and 4-13 below. Compared to the supply chain performance without coordination, both RSLRD and RSRWPC result in better supply chain performance. RSLRD results in 5.51% higher supply chain profits and 4.10%, 40.37%, 2.93% for the efficiencies of supplier, manufacturer, and wholesaler. RSRWPC results in 8.41% higher profits and 6.88%, 45.67%, 6.21% for the efficiencies of supplier, manufacturer, and wholesaler. In Scenario II, the effect of the wholesale price discount, r_2, reflects the significantly lowered loan rate discount from Scenario I. After comparison, it is concluded that RSRWPC brings the best supply chain performance.

Table 4-12: Supply Chain Profit Comparisons

Scenario	π_{sc}	π_1	π_2	π_3	$\theta_{1\to 2}$	$r_{1-\text{I}}$	$r_{1-\text{II}}$	r_2
NO Coordination	$663,778.94	$610,877.23	$26,558.12	$26,343.59	N/A	N/A	N/A	N/A
RSLRD	$700,344.06	$635,947.61	$37,279.69	$27,116.76	0.18	42.61%	N/A	N/A
RSRWP	$719,596.84	$652,930.62	$38,686.44	$27,979.78	0.19	N/A	25.48%	90.64%

Table 4-13: Supply Chain Efficiency Comparisons

Scenario	η_{sc}	η_1	η_2	η_3	$\theta_{1\to 2}$	$r_{1-\text{I}}$	$r_{1-\text{II}}$	r_2
NO Coordination	90.59%	86.36%	64.44%	88.09%	N/A	N/A	N/A	N/A
RSLRD	95.07%	89.91%	90.46%	90.67%	0.18	42.61%	N/A	N/A
RSRWP	98.21%	92.31%	93.87%	93.56%	0.19	N/A	25.48%	90.64%

Chapter 5: Conclusions

Supply chain financing is an emerging field in the area of supply chain management with research coming from many different perspectives such as trade credit and internal and external direct financing. Exploring effective financing methods is necessary for supply chains to facilitate their normal operations and maximize profits. However, the existence of the greedy actions of supply chain members can lead to suboptimal supply chain performance. To optimize overall supply chain performance as well as align the operations and objectives of different supply chain members, appropriate supply chain coordination mechanisms are proposed. The ultimate goal is to increase the profit of the supply chain as a whole while also benefitting all the supply chain participants.

The initial work focuses on comparing profitability of a two-echelon supply chain using third-party financing and supplier-led financing under various demand risk levels. The research conclusion and managerial insights are generated based on classic Newsvendor model assumptions, with added assumptions regarding the financing parameters. It is assumed that the interest rate charged by the supplier is stochastic but has an upper bound which is less than that charged by third-party financial institution. The supplier is assumed to have the option to invest into the market at a normally distributed rate with a mean value equal to that charged by third party institution. Of particular interest is that, in most cases, the retailer will always choose to finance from the supplier to order a larger quantity at a relatively low interest rate, however, the supplier may not lend the available working capital, preferring to invest in the market, particularly when the market investment rate is much greater than the loan rate. The supply chain also typically prefers supplier financing to third party financing assuming optimal amount of capital is borrowed, which opens path to the research of two-echelon supply chain coordination with internal (supplier) financing. This work brings managerial insights to supply chain entities on the effects of different types of financing under demand risks. Generalized conclusions about the supplier, retailer and supply chain are drawn which illustrate the optimal ordering quantities and loan principals under various scenarios of demand risk and interest rates.

Modified Wholesale Price Contract (MWPC) is then applied to coordinate the two-echelon supply chain with internal financing. The profits and efficiencies of all entities are compared between a traditional Newsvendor model, MWPC, and traditional wholesale price contract (TWPC). In a traditional Newsvendor model, the overall supply chain performs better in retailer-led Stackelberg than in supplier-led. However, the retailer, supplier and overall supply chain do not yield the greatest profit in the traditional Newsvendor case. With implementing TWPC, the retailer, supplier and overall supply chain performance is improved with increased profits and efficiencies. However, resource disparity exists between supply chain entities. With implementing the proposed MWPC, the supply chain efficiencies reach as high as 98.30% in both retailer-led and supplier-led Stackelberg with relatively evenly distributed resource between the capital constrained retailer and the supplier. It is concluded that the proposed MWPC works the best in coordinating two-echelon supply chain financing problem. Proper contract parameter setting can guarantee the increased profits and efficiencies for both the retailer and supplier. This work comprehensively combines supply chain coordination with internal financing, which not only enriches the theoretic development in the field of supply chain management, but also brings application value to the industry.

This research is the first to study the coordination of a multi-echelon multi-period supply chain financing. Mathematical methodologies are applied to construct the profit and efficiency models for supply chain entities as well as the overall supply chain, where the value of cash flows are incorporated into the financing models. The key problem to solve revolves around how to determine the optimal contract periods to maximize the overall supply chain profit, and how to coordinate various supply chain individual actions under the determined contract period. There are two coordination mechanisms studied and the proposed Revenue Sharing combing Revised Wholesale Price Contract (RSRWPC) results in a greater coordination effect for the multi-echelon multi-period supply chain finance problem than a simple revenue sharing contract. RSRWPC's successful coordination is due to two aspects. First, it benefits the overall supply chain through increasing the entire supply chain profit. Secondly, it further improves each supply chain member's individual efficiency, which is a key requirement for convincing the supply chain entities to implement a contract mechanism. Different contract parameter settings bring different coordination results, which were demonstrated in the numerical study. Therefore, contract parameter values should be carefully determined in order to bring the increased profits and efficiencies for supply chain entities and the entire chain.

For future work, it would be interesting to explore other contract mechanisms to coordinate the similar supply chain problems. Also the proposed RSRWPC can be analyzed for different supply chain settings such as a nonlinear structure, price-dependent market demand, and different service level requirements or product line variety. Future extensions can also include supply chain network setting, risk –averse entities, as well as fixed period financing.

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Appendix 1. Proof for Lemma 5

Step 1. Based on Equation 3.14, the first derivative of retailer profit to order quantity is calculated:

$$\frac{d(\pi_{r-III})}{d(Q_{III})} = s_r - c_r + g_r - c_r i_\gamma t - (s_r - v_r + g_r)F(Q_{III}), where\ F(Q_{III})\ is\ the\ cdf\ of\ Q_{III}.$$

When $v_r < c_r(1+i_\gamma t)$, $s_r - v_r + g_r > s_r - c_r + g_r - c_r i_\gamma t$.

Then, $0 < \frac{s_r - c_r + g_r - c_r i_\gamma t}{s_r - v_r + g_r} < 1$

Therefore, Q^*_{III} exists and is calculated by making $\frac{d(\pi_{r-III})}{d(Q_{III})}$ equal 0:

Step 2. $$\frac{d^2_{\pi_{r-III}}}{dQ_{III}{}^2} = -(s_r - v_r + g_r)f(Q_{III}) < 0$$

The second derivative of π_{r-III} is negative indicating the concavity of the function. When $Q_{III} < \mathrm{Q}^*_{\mathrm{III}}$, π_{r-III} is monotonically increasing in order quantity Q_{III} while decreasing when $Q_{III} \geq \mathrm{Q}^*_{\mathrm{III}}$. Therefore, the optimal Q^*_{III} has a unique value.

Step 3. However, when $v_r \geq c_r\ (1+i_\gamma t)$,

$\frac{d(\pi_{r-III})}{d(Q_{III})} = s_r - c_r + g_r - c_r i_\gamma t - (s_r - v_r + g_r)F(Q_{III}) > 0$, indicating

π_{r-III} is monotonically increasing in Q_{III}.

Appendix 2. Proof for Proposition 6

In order to investigate the effect on profits with increasing demand, two risk levels are used: Risk level A and Risk level B. Assume that $\mu_A = \mu_B$ and $\sigma_A < \sigma_B$, so that the risk is greater in risk level B and therefore the effect of ΔD, $(D_A - D_B)$ is assumed small.

- Demand risk level A: $\frac{\sigma_A}{\mu_A}$, the demand D_A, optimal order quantity ${Q_A}^*$, the retailer's profit π_{r_A} , the supplier's profit π_{s_A} , the supply chain's profit π_{sc_A}.
- Demand risk level B: $\frac{\sigma_B}{\mu_B}$, the demand D_B, optimal order quantity ${Q_B}^*$, the retailer's profit π_{r_B}, the supplier's profit π_{s_B}, the supply chain's profit π_{sc_B}.

Assuming $\frac{\sigma_A}{\mu_A} < \frac{\sigma_B}{\mu_B}$, then ${Q_A}^* < {Q_B}^*$ when $R_\gamma = M_\gamma^*$; while ${Q_A}^* = {Q_B}^* = \frac{R_\gamma + M_\partial}{c_r}$ when $R_\gamma < M_\gamma^*$.

3) When the financing amount from the supplier is optimal ($R_\gamma = M_\gamma^*$), the retailer will have enough capital to order from supplier, $Q \geq D$.

$$\pi_{r\_A} = s_r D_A - v_r(D_A - {Q_A}^*) - c_r {Q_A}^* - {Q_A}^* c_r i_\gamma t + M_\partial i_\gamma t$$

$$\pi_{r\_B} = s_r D_B - v_r(D_B - {Q_B}^*) - c_r {Q_B}^* - {Q_B}^* c_r i_\gamma t + M_\partial i_\gamma t$$

$$\pi_{s\_A} = (s_s - c_s){Q_A}^* - (i_\varepsilon - i_\gamma) c_r {Q_A}^* + (i_\varepsilon - i_\gamma) M_\partial$$

$$\pi_{s\_B} = (s_s - c_s){Q_B}^* - (i_\varepsilon - i_\gamma) c_r {Q_B}^* + (i_\varepsilon - i_\gamma) M_\partial$$

Therefore,

$\pi_{s\_A} - \pi_{s\_B} = (s_s - c_s - (i_\varepsilon - i_\gamma)c_r)({Q_A}^* - {Q_B}^*) < 0$, indicating supplier profit increases with risk level.

$\pi_{r\_A} - \pi_{r\_B} = (v_r - c_r - c_r i_\gamma t)({Q_A}^* - {Q_B}^*) + (s_r - v_r)\Delta D$, indicating there is no trend to be identified for retailer profit.

$\pi_{sc\_1} - \pi_{sc\_2} = (v_r - c_r - c_r i_\gamma t + s_s - c_s - (i_\varepsilon - i_\gamma)c_r)({Q_A}^* - {Q_B}^*) + (s_r - v_r)\Delta D$, indicating there is no trend to be identified for supply chain profit.

4) When the financing amount from third party institution is sub-optimal ($R_\gamma < M_\gamma^*$), retailer will not have enough capital to order from supplier, $Q < D$.

$$\pi_{r\_A} - \pi_{r\_B} = (s_r - c_r + g_r - c_r i_\gamma t)({Q_A}^* - {Q_B}^*) - g_r\Delta D = -g_r\Delta D$$

$$\pi_{s\_A} - \pi_{s\_B} = (s_s - c_s + g_s - (i_\varepsilon - i_\gamma)c_r)({Q_A}^* - {Q_B}^*) - g_s\Delta D = -g_s\Delta D$$

$$\pi_{sc\_A} - \pi_{sc_B} = -(g_r + g_s)\Delta D$$

Given the nature of ΔD, it is concluded that under this scenario of sub-optimal financing from third financial institution, the profits of the retailer and supplier are not sensitive to the changing demand risk levels while supply chain profit has a relatively obvious but not identifiable trend.

Appendix 3. Proof for Lemma 7

Step 1. Let $j(p_S) = p_R + f_2(p_S) - f_1(p_S) - f_1(p_S)i_R$,

$$h(p_S) = p_R + f_2(p_S) - f_3(p_S)$$

$$w = \frac{g(p_S)}{h(p_S)}$$

The following proving process is based on Eq. (3.24), Eq. (3.26), and Eq. (3.27).

From $i_R < \frac{2(p_R - c_R)}{c_R}$, it is inferred that $i_R c_R < 2(p_R - c_R)$, Then $i_R c_R < p_R - c_R + g_R$.

So $j(p_S) > 0$;

Since $p_R > v_R$, then $p_R + g_R > v_R$, $h(p_S) > 0$ and thus, $w > 0$.

One of the model assumptions is that $v_R < c_R$, which follows $-v_R > -(c_R + c_R i_R)$,

and $p_R + g_R - v_R > p_R + g_R - (c_R + c_R i_R)$. Therefore, $w = \frac{j(p_S)}{h(p_S)} < 1$.

According to the nature of the normal distribution, $Q_R^*(i_R, p_S)$ exists when $0 < w < 1$.

Step 2. $$\frac{d^2_{\pi_R(Q, i_R, p_S)}}{dQ^2} = (v_R - p_R - g_R)f(Q) < 0$$

The second derivative of $\pi_R(Q, i_R, p_S)$ is negative indicating the concavity of the objective function. When $Q < Q_R^*$, π_R is monotonically increasing in order quantity Q while decreasing when $Q > Q_R^*$. Therefore, the optimal $Q_R^*(i_R, p_S)$ has a unique value.

Appendix 4. Proof for Lemma 8

$$\text{Take } k(p_S) = (p_S - c_S) + (i_R - 2i_\epsilon)f_1(p_S) + f_4(p_S)$$

$$u = \frac{(p_S - c_S) + (i_R - 2i_\epsilon)f_1(p_S) + f_4(p_S)}{f_4(p_S)}.$$

The following proving process is based on Eq. (3.24) and Eq. (3.28).

If $\frac{p_S - c_S}{c_R} < 2i_\epsilon - i_R$, then $p_S - c_S - c_R(2i_\epsilon - i_R) < 0$, and $p_S - c_S - c_R(2i_\epsilon - i_R) + g_s < g_s$, therefore $u < 1$

If $2i_\epsilon - i_R < \frac{2(p_S - c_S)}{c_R}$, then $2i_\epsilon - i_R < \frac{p_S - c_S + g_s}{c_R}$, and $(p_S - c_S) + (i_R - 2i_\epsilon)c_R + g_s < 0$, therefore $u > 0$

According to the nature of the cumulative density function for normal distribution, $0 < u < 1$ is proven to guarantee the existence of the Q_S^*. Concavity is verified by $\frac{d^2_{\pi_S(Q, i_R, p_S)}}{dQ^2} = -g_R f(Q) < 0$.

Appendix 5. Proof for Lemma 9

Let $m(\mathrm{p_S}) = p_R + (p_S - c_S) + f_2(p_S) + f_4(p_S) - (1 + 2i_\epsilon)f_1(p_S)$,

$$n(\mathrm{p_S}) = p_R + f_2(p_S) + f_4(p_S) - f_3(p_S); \; y = \frac{m(p_S)}{n(p_S)}.$$

The following proving process is based on Eq. (3.24), Eq. (3.26), and Eq. (3.28):

If $i_\epsilon \le \frac{p_R - c_R}{c_R} + \frac{p_S - c_S}{c_R}$, then $1 + 2i_\epsilon \le 1 + \frac{2(p_R - c_R) + 2(p_S - c_S)}{c_R} < \frac{p_S - c_S + p_R + g_R + g_S}{c_R}$, so

$$m(p_S) > 0$$

$$p_R + g_R + g_S - v_R > 0, \text{ so } n(p_S) > 0; \text{ thus } y > 0$$

If $i_\epsilon > \frac{p_S - c_S}{2c_R} + \frac{v_R - c_R}{2c_R}$, then $1 + 2i_\epsilon > \frac{p_S - c_S + v_R}{c_R}$. Therefore, $(1 + 2i_\epsilon)c_R > p_S - c_S + v_R$.

Thus $m(p_S) < n(p_S)$ and $y < 1$ is proven.

Satisfying the nature of the cumulative density function for normal distribution ($0 < y < 1$) guarantees the existence of the Q^*_{SC}. Next step is to check the concavity of the objective function:

$\frac{d^2_{\pi_S(Q, i_R, p_S)}}{dQ^2} = -(g_R + p_R + g_S - v_R)f(Q) < 0$, which confirms the concavity. So Q^*_{SC} is unique.

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Appendix 6. Q'_{jt} Values Resulting from Non-Cooperative SC

Q'_{31}	Q'_{32}	Q'_{33}	Q'_{34}	Q'_{35}	Q'_{36}	Q'_{37}	Q'_{21}	Q'_{22}	Q'_{23}	$Q_{\prime 24}$
1000	850	520	386	264	730	650	1000	1050	320	465
Q'_{25}	Q'_{26}	Q'_{27}	Q'_{11}	Q'_{12}	Q'_{13}	Q'_{14}	Q'_{15}	Q'_{16}	Q'_{17}	
309	600	510	1500	550	520	300	286	1310	0	

Appendix 7. Q_{jt} Values Resulting from Coordination Effects in Scenario I

Q_{31}	Q_{32}	Q_{33}	Q_{34}	Q_{35}	Q_{36}	Q_{37}	Q_{21}	Q_{22}	Q_{23}	Q_{24}
924	1140	377	225	222	433	962	1020	1020	1000	0
833	1181	735	521	290	200	2907	1020	1100	914	610
711	984	324	0	1013	606	285	818	1000	576	0
602	1396	862	199	773	121	1699	1020	1000	1000	720
1020	1303	699	407	133	336	616	1020	1488	835	839

Q_{25}	Q_{26}	Q_{27}	Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{15}	Q_{16}	Q_{17}
1020	0	223	1500	1000	2000	0	1000	0	0
1322	0	1701	1620	1500	614	1110	1022	116	685
1047	546	0	1000	1200	404	0	1000	622	0
654	221	1137	1120	1100	1000	684	780	1221	847
0	0	432	1500	1600	1035	186	0	0	293

Appendix 8. Q_{jt} Values Resulting from Coordination Effects in Scenario II

Q_{31}	Q_{32}	Q_{33}	Q_{34}	Q_{35}	Q_{36}	Q_{37}	Q_{21}	Q_{22}	Q_{23}	Q_{24}
718	2020	893	316	131	71	602	1038	2000	1093	206
1020	1172	509	11	616	260	441	1320	1265	876	0
687	1069	960	756	113	243	497	1087	819	1060	506
243	648	990	595	1305	1206	1075	1013	108	760	1005
513	624	450	1760	1800	479	809	1023	114	900	1310
507	524	1020	1055	454	420	458	1027	504	520	1300
776	1301	409	968	1058	420	731	1000	1121	365	1308
1074	1217	906	600	1417	1654	703	1220	1071	1260	246
1020	665	965	1100	412	1000	459	1300	385	1456	609
1373	875	714	1025	778	746	366	1603	645	1000	739

Q_{25}	Q_{26}	Q_{27}	Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{15}	Q_{16}	Q_{17}
301	0	127	1378	1700	1253	116	300	0	18
229	160	168	1680	1576	680	0	127	60	0
413	43	397	1327	1000	639	1006	113	0	240
625	1756	525	1453	0	428	1285	645	1956	25
1900	779	409	1403	0	1034	1010	2000	929	159
209	829	49	1577	0	924	1000	59	1020	0
718	670	481	1100	1561	0	1133	1018	350	501
1689	1800	557	1500	791	1720	0	1475	2000	357
840	772	259	1500	185	1700	365	1240	570	61
1108	537	245	1900	348	1200	609	1300	398	122