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Reliability modeling and optimization of new product development (NPD) process

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A dissertation submitted to the Faculty of
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My Wonderful Wife, Afshan Roshani

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Abstract

The development of complex engineering products has various challenges in terms of meeting the development budget, release time, and performance goals. Most of the new product development processes have been experiencing challenges to meet these goals within an increasingly competitive global market environment. In addition, the reliability of the new product has always been a matter of concern for new product developers and it has been a challenge to balance the development cost and time while increasing product reliability

The main goal of this research is to develop methodologies to support new product developers meet their product requirements and strategic business goals. This research consists of two main parts. In the first part, a novel methodology in modeling and optimizing the reliability growth is proposed, which considers the multiple stages of a new product development process and provides an optimal reliability growth plan in terms of time, cost, and the reliability of the newly developed products. The proposed approach differs from the existing research in the literature by, (1) considering multiple stages of the reliability growth program in the early stages of the new product development process, and (2) optimizing all three new product development goals simultaneously. The second part of this research provides an innovative approach to model and optimize the planning of a verification and validation (V&V) process in the early stages when designing a new product. This mathematical approach provides an optimal way of implementing a design verification and validation process to have maximum reliability improvement of a new product under development, when the implementation time and cost are limited.

Keywords: Reliability Improvement, New Product Development (NPD), Reliability Growth Planning (RGP), Verification and Validation (V&V), Optimization, Decision Making

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Chapter 1. Introduction

1.1. Background

Developing and releasing new products is one of the most important sources of revenue for any organization because it brings higher revenues, increase customer loyalty, and ultimately higher profits [1, 2, 3, 4]. New products also provide various benefits to a firm, its shareholders, and its employees [5, 6]. The continuously increasing budget for new product development (NPD) in world-class companies shows the importance of an effective and timely NPD process to stay ahead of the competition [1]. As presented in Figure 1.1, the NPD process starts with a new concept for a product or a system. After identifying and defining the product requirements in conceptual and detailed design stages, small number of pilot and prototype products are built and tested for performance and function verification [7]. Mass production then proceeds following the verified product design objectives and requirements.

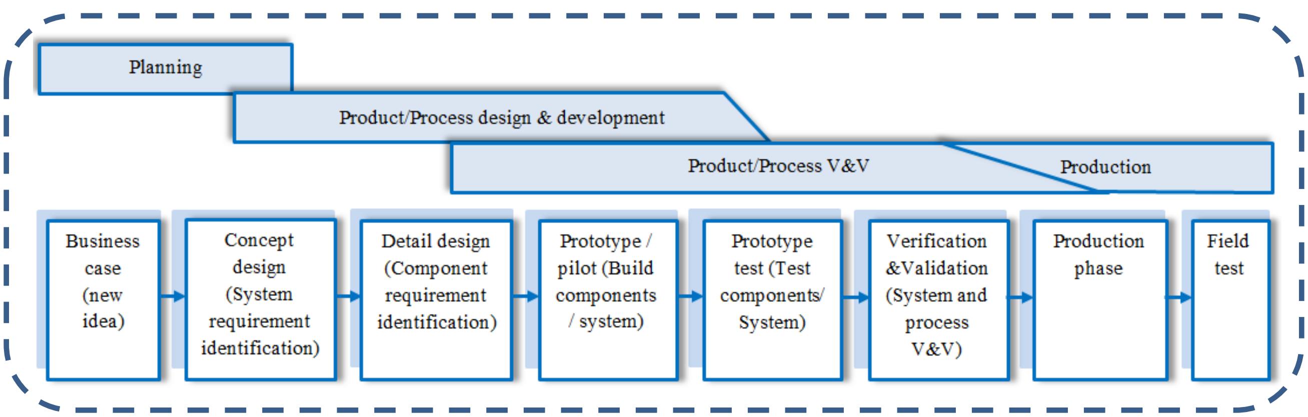

Figure 1.1: New product development process

Product reliability has always been considered as a major performance metric in the NPD process. Several reliability engineering/management activities, including reliability prediction, modeling, and optimization, occur during the development of a new product in order to improve the reliability of the product. Reliability growth is a part of reliability management program and is performed as one of the processes toward improving the reliability of a product with time. The basic elements for reliability growth of a newly developed product are: defining the reliability requirement goal (target), estimating the reliability of current design, evaluate the required reliability improvement by comparing current design reliability and reliability target, establishment of the most effective approach to reliability growth achievement, execution of the improvement activities, and improvement validation [8].

As one of the main reliability improvement processes in the NPD reliability management program, the process for reliability growth of a new product usually consists of two main stages: 1) reliability analysis and improvement in the design phase and through a reliability growth test phase, and 2) reliability analysis after the products are developed through the field failure data analysis [7]. Considering reliability growth at the design stage provides multiple technical, scheduling, and budgeting advantages such as: 1) the essential product modifications for reliability improvement in the early product design phases are easy to implement, even if they are of a greater magnitude; 2) selecting more reliable components is more flexible since the parts listed, i.e. bill of materials (BOM), are not finalized; 3) the design modifications can be made before the parts are ordered, even those with the longer lead times; 4) the reliability of the final product would be higher and it requires considerably shorter testing time for additional reliability growth; 5) fewer test failures result in less delays in the test schedule due to failure investigations and their resolutions [7]. The reliability growth process can also be applied to the new developed products after they are developed and fielded. It usually happens when the field failure data analysis results show the need for the product reliability improvement.

It has been proven in the literature, as well as in most industrial practice, that the reliability growth program for reliability improvement should be considered as an essential part of an overall reliability management program in the early stages when developing a new product. This is particularly true for a design that uses new or unproven technologies, components, or a substantial content of software. In this case, the NPD process may expose, over a period of time, many types of failures having design related causes. It is crucial to decrease the probability of failure due to these weaknesses to the greatest extent possible to prevent their future occurrence while there are in the field. At that late stage, design correction is extremely inconvenient, expensive, and time consuming. As it is mentioned in the “International Electrotechnical Commission (IEC) IEEE 61014 standard, programmes for reliability growth,” [7] the reliability design analysis is very important, as it allows early identification of potential design weakness before design completion. Based on the aforementioned reasons, this research is focused on the reliability analyses of a new developed product during the design stages of the new product development process.

When developing modern complex engineering systems, the reliability growth planning (RGP) becomes critical to providing management with decision support in terms of the timing of a new product release, reliability performance monitoring and prediction, and budgeting product development cost. An accurate and realistic reliability growth plan can provide trade-off information between the product development schedule, the program budget, and achievable reliability level of a new product. Therefore, RGP is an essential part of a reliability management program in the early stages of developing a new product. RGP is increasingly important when developing complex engineering systems such as aerospace systems, modern vehicle systems, and aviation systems, since there usually are multiple development stages and complex performance verification and validation activities in such complex engineering systems.

The existing RGP models in the literature emphasize the reliability growth modeling by utilizing the actual test data, which is under the assumption that initial reliability growth testing data are available. However, in the early product development stages, such as in the concept design stage and product portfolio selection stage, actual testing data is rarely available for RGP modeling. In addition, most existing RGP models consider a single product development stage by optimizing the single reliability maximization objective, which may not be able to effectively model the different reliability growth profiles over a product's multiple development stages.

In this research, the multiple stages RGP model for early stages of new product development process is investigated. A multi-stage RGP, which is able to model each stage's unique reliability growth profile due to different growth rates, new contents/technology, time and budget allocation for that specific stage is proposed. In addition to maximizing the reliability objective, minimizing the projected total reliability growth testing time and consequently minimizing the projected reliability testing cost are also considered. The new multi-stage and multi-objective RGP enables decision makers to intelligently allocate limited and expensive testing units/subsystems and testing time to each individual development stage. As a result, the optimality can be achieved in terms of reliability, program schedule, and product development cost.

One of the most important processes in improving the reliability of a product is the product design verification and validation (V&V) process, which consists of defining and implementing various engineering activities, such as product simulation, performance testing, engineering analytics, and product modifications, designed to confirm that a product meets its respective specifications and fulfills its intended functions [2]. In other words, achieving a certain level of product reliability through a reliability management program, which is one of the new product design objectives, can be accomplished by implementing product verification and validation activities. The product verification and validation process, as an integral part of the NPD process, needs to be adequately deployed during the early product development stages for reducing the product development cost and cycle. The planned V&V activities should be conducted on schedule, in a specific sequence, and under a limited budget. In addition, the product itself is usually highly complex and its failures can be attributed to different failure modes. The product complexity, multiple and possible common and random failure modes, and various V&V options along with the demanding design requirements and objectives, call for a cost and time effective V&V activity plan which optimally covers all major failure modes of the product. From the reviewed literature, no effective and quantitative methodologies have been thoroughly explored for optimal planning of V&V activities.

In addition to multi-stage RGP modeling and optimization, a general mathematical modeling and optimization approach will be proposed to plan the verification and validation process. Reliability of the new developed product is considered as the objective function which has to be maximized by implementing a set of V&V activities, which should cover different failure modes obtained from the design failure mode and effect analysis (DFMEA). In addition to considering the reliability of the new developed product, the proposed design V&V modelling and optimization approach also considers the limited time and budget of the new product development process. Sequencing of V&V activities and their effectiveness are also important factors which are considered in the proposed design V&V activities planning model.

1.2. Research motivation

It is reported from the literature that a product or system development process by itself is a complex process; arguably even more complex than the system it produces [4]. The structure of the product development process impacts a project's cost and schedule risk [1, 5]. Despite decades of industrial experience, it is found that designing and developing increasingly complex products, e.g., aerospace products, still incurs significant cost overruns [6, 9]. Recent lessons from the NPD projects of major aerospace industries in the United States show that NPD processes are mostly plagued with cost overruns, schedule delays, and quality issues during design stages. Some examples are summarized in Table 1.1.

Table 1.1: Examples of NDP delays, cost overruns, and quality issues

Product	Company	Issues	Source
787 Dreamliner	Boeing Co	Delays due to shortages of key materials and slow deliveries by suppliers and also structural flaws.	The Wall Street Journal (2007 [10] and 2009 [11])
Chevy Volt	General Motors	Cost overrun during design.	CNN Money (2009) [12]
The Honda/GE HF120 turbofan engine	Honda	Design issues and an unanticipated test program glitch; a part of the gearbox failed during the test.	Flying (2013) [13]
F-35	United Technologies Corp.'s Pratt and Whitney unit	Delays in delivering engines; quality flaws and technical issues; systemic issues and manufacturing quality escapes.	Defence-aerospace.com Bloomberg Business (2014) [14]
Sikorsky	US Marine Corps' (USMC's)	A failure in the main gear box and need for redesign of the component; design problems with wiring and hydraulics systems; budget constraints.	HIS Jane's 360 (2015) [15]

NASA's main projects, including the international space station and the NASA Ares-I launch system, also faced cost overruns and time delays [16, 17]. As it was reported in 1999 [18], the international space station project prime contract had grown 25%, from $783 million to $989 million. Historically, large NASA programs have cost overruns by 50% [19]. In addition, the United States Department of Defense (DoD) development programs are mostly plagued with cost overruns and schedule delays [20]. Studying a set of 96 major new weapon systems development programs in the United State DoD, presented in [21] in 2009, showed an average development cost growth of 42% and an average delay of 22 months. It is also reported in [21] that almost 50% of the DoD's major defense acquisition programs do not meet cost performance goals. In addition, 80% of programs have experienced an increase in unit costs from initial estimates. Another study [22] of similar programs in the 1970s and 1980s showed overruns of 50% in cost and 33% in schedule. A defense science board task force report (2008) [23] mentioned a major increase in the number of military systems that are being recognized as not operating sufficiently, when the main issue is reported as poor reliability.

It is unlikely that large scale products will compete solely on the basis of technical performance. What will differentiate such systems, and their developers, is the ability to balance all the dimensions of product development performance, including product reliability, and product development cost and time. The above discussed issues show the lack of effective methods for the NPD process, which simultaneously optimizes process cost, time, and reliability when developing large scale complex engineering systems. This gap is an emerging field of research in the new product development process literature. This research provides mathematical approaches to model and optimize the two major reliability improvement processes in the NPD process, including reliability growth planning and product design verification and validation planning, which can significantly reduce new product development process time, cost, and, at the same time, increase the reliability of the developed product.

1.3. Research objectives and contributions

The first main objective of this research is to explore new reliability growth planning methods through a multi-stage multi-objective reliability growth planning (MS-MO-RGP) model in early stages of developing a new product. Multiple objectives of developing a new product, including the product development schedule, program budget, and achievable reliability level of a new product are considered when developing the MS-MO-RGP model. The developed model is solved using an integrated multi-objective optimization algorithm.

The second main objective of this research is to investigate a new mathematical modeling and optimization approach for planning a product design verification and validation process, which is a part of the reliability management program. Different characteristics of design V&V activities implementation such as duration and cost of V&V activities, sequence and effectiveness of implementing V&V activities, and product reliability improvement after implementing V&V activities will be considered in the V&V activities planning model. The result of the V&V activities planning process would be a set of V&V activities that can be implemented during the product development process to optimize the reliability of the product considering the limited NPD budget and cost.

1.4.Organization of the rest of the dissertation

The remainder of the dissertation is organized as follows:

Chapter 2 provides a literature review of the reliability growth models as well as the verification and validation planning models which have been used in the new product development stages. The multi-objective multi-stage reliability growth planning (MO-MS-RGP) to model and optimize one of the main reliability improvement processes in new product development process, is presented in **Chapter 3**. It provides the multi-objective multi-stage reliability growth planning of the new developed product. The multi-objective and multi-stage RGP is formulated and an extended genetic algorithm is introduced which provides a set of optimal solutions of the RGP problem in the form of Pareto frontiers. In order to reduce the size of Pareto optimal solutions, the data envelopment analysis (DEA) approach is proposed for Pareto solution reduction. The proposed multi-objective and multi-stage reliability growth planning model is applied to the example of next generation dual fuel engine development. Guidelines and insights about the RGP model and trade-off analyses for Pareto solutions are also presented in this chapter. **Chapter 4** presents the proposed approach to model and optimize the verification and validation (V&V) activities planning process, as another important reliability improvement process during developing a new product. It starts with the needs for a verification and validation planning process and the challenges in modeling and optimizing this problem. The proposed mathematical model of V&V activities planning with the objective of maximizing the product reliability improvement, considering limited time and budget of new product development, as well as, failure coverage and V&V activities sequencing constraints, is discussed in this chapter. The proposed V&V activities planning method is applied in a case study of the next generation engine development process. **Chapter 5** concludes this thesis by providing conclusions and discussion of future research areas in reliability growth planning of a new developed product and V&V activities planning process.

Chapter 2. Literature Review and Background Information

2.1. Background

Reliability is defined as the ability of an item to function under given conditions for a period of time [24, 25]. Reliability of a product has been always a matter of concern when developing a new product. Concerning the reliability of a new product, the new product development (NPD) process covers the time frame from the reliability requirements review, conceptual design, preliminary and detail design, pre-production, product reliability testing, conventional or accelerated testing, such as reliability growth testing or accelerated life testing, verification and validation (V&V) activities, production with screening tests, and on-going reliability tests. Product reliability improvement can also be considered during the fielded period, i.e., use or deployment period, as a part of the NPD process [26]. However, it has been proven in the literature that considering reliability before the fielded period is more effective in terms of cost and duration of product modifications [3].

The main reliability improvement activities during early stages of developing a new product include: 1) design failure mode and effect analysis (FFMEA), 2) reliability growth planning and testing, and 3) verification and validation (V&V) activities planning and implementation. In a program of reliability improvement during the early stages of a product design, the design is evaluated to determine whether its components and sub-systems or their interactions, i.e. interfaces, constitute potential failures when subjected to the operational and environmental stresses [27]. The design failure mode and effect analysis (DFMEA) process is implemented in this phase to identify the potential failure modes and their level of criticality, which is usually measured by the risk priority number (RPN). The outcomes of conducting DFMEA can be a list of verification and validation (V&V) activities which can be implemented to mitigate the design failure modes. V&V activities include analytical engineering activities such as testing, simulation, and modification activities designed to improve the product and meet its technical, functional, and regulatory requirements. In other words, by implementing the V&V activities, the product failure modes and their causes are addressed through design modifications and improvement, and the reliability of new design is re-assessed. Results of the design analysis are compared with the product reliability goals or requirements, and necessary modifications are made for the possible improvements. Meanwhile, reliability growth, which is defined as the improvement in the reliability of a product over a period of time, is monitored and the progress is recorded. The reliability growth planning (RGP) includes the process of making decision about the number of essential test units, the duration of test, and also estimating the amount of product reliability improvement considering limited time and budget. After planning the reliability growth, the reliability growth test (RGT) will start.

This research is based on the reliability improvement activities during the design stages of developing a new product. It investigates a reliability growth modeling and optimization during the early stages of the new product development process as well as the modeling and optimization of the product verification and validation (V&V) process planning. The V&V process happens after design failure mode and effect analysis (DFMEA) and as an integrated process in the reliability growth program. Therefore, **Section 2.2** of this chapter sketches the related models for reliability growth modeling. In addition, a brief literature review of the design verification and validation process is provided in **Section 2.3**.

2.2. Reliability Growth Planning (RGP)

Reliability growth is defined as the improvement in the reliability of a product (component, subsystem, or system) over a period of time due to modifications in the product's design and/or manufacturing process [7, 24, 25]. The reliability growth principles of a new developed product are usually the same during design and test stages since both include identifying and mitigating or removing failures to improve the product reliability. Both measure that improvement by making the comparisons between the estimated reliability and the reliability goal. Most mathematical models for the reliability growth have been proposed and applied to estimate the achieved growth and the projected reliability. Reliability growth models support the planning of the reliability management program by providing the estimation of the number and the magnitude of the changes during the design and development process, or the test time required to reach a specified reliability goal [3].

In general, the reliability growth can be divided into three different areas of planning, tracking, and projection [24, 25]. Reliability growth tracking and projection models have generally been established to evaluate the reliability of the system considering specific assumptions about test situation, data collection, and the way that corrective actions (CAs) are implemented. Tracking models only consider the failure data collected during developmental testing in order to approximately estimate the reliability improvement during the test. In other words, the reliability growth tracking focuses on the analysis of a system's current reliability. The growth projection models focus on estimating system reliability following implementation of corrective actions (CAs) to mitigate the identified failure modes. Traditional emphasis on reliability growth has been on tracking and projection models, while recent focuses have, as well as the focus of this research, have been on reliability growth planning. Reliability growth planning (RGP) models are mostly extended versions of the assessment models that can be applied to plan an appropriate reliability growth program before the system level reliability assessments are available. Each of these areas of reliability growth has been applied to complex systems with continuous failure data are continuous, as well as to complex systems whose failure data are discrete. Complex systems whose test durations are discrete are referred to as one-shot systems such as guns, rockets, and missile systems. Continuous systems are systems whose usage is measured in the continuous time domain, such as hours or miles. A great deal of research has been done over the past several decades in each of these areas. A comprehensive literature review of existing reliability models is provided in [24, 25].

Reliability growth planning (RGP) is an approach to optimizing testing resources, quantifying potential risks, and developing planning curves that reflect the successful achievement of reliability goals. The RGP results support development of the overall system test planning and support decision makers to balance time, cost, and risk. RGP requires an understanding of the system reliability assessment, system reliability requirements and goals, and the total planned testing times and cost. The reliability growth planning (RGP) models are mostly formulated in terms of the failure rate (or intensity), or probability of survival to a specified time (reliability). To provide a reliability growth planning model, several inputs are required, e.g., the initial reliability estimation, the goal reliability, the duration of design or test period. These inputs can be estimated using the historical data from previous test results on a similar product to plan and predict the future reliability growth program, provided that the use and test conditions are similar [3, 26]. The basic assumption for reliability growth is that the product development team makes design changes to correct any discovered failure modes during product development and testing. Reliability growth was first observed and modeled by Duane based on the learning curve properties [3]. Built on Duane's basic model, more advanced reliability growth models have been developed for the purposes of quantifying reliability estimation uncertainties, modeling specific system applications, e.g., software reliability growth in design and development [28, 29], and single mission (one-shot) system reliability growth [30, 31, 32]. In the following sub-sections, the most commonly used RGP models are introduced and a literature review of recently applied RGP models is provided.

2.2.1. Duane Reliability Growth Model

J. T. Duane (1964) [33] plotted failure data on a log-log scale from several systems under reliability growth testing and observed that the cumulative failure rate is approximately linearly decreasing over the accumulated testing time. Denote $N(t)$ as the cumulative number of failures up to time t during the reliability growth testing, the cumulative failure rate is $C(t) = N(t)/t$ [33]. Based on the learning curve characteristic, Duane's empirical observations can mathematically be expressed as: $ln[C(t)] = \delta - \alpha\, ln(t)$, where $\delta, \alpha > 0$. Duane interpreted the parameter α as the "Growth Rate". The log-linear relationship can be rewritten as: $C(t) = \lambda t^{-\alpha}$, where $\lambda = e^{\delta}$. Since $C(t) = N(t)/t$, the cumulative number of failures by time t is: $N(t) = \lambda t^{(1-\alpha)}$. Thus, the instantaneous failure rate $r(t)$ at time t is: $r(t) = \frac{d}{d(t)}[N(t)] = \lambda(1-\alpha)t^{-\alpha}$. The MTBF is: $M(t) = \frac{1}{r(t)} = [\lambda(1-\alpha)t^{-\alpha}]^{-1}$ [3].

2.2.2. Crow (AMSAA) reliability growth model

L. H. Crow, in 1974 [34], considered the power law reliability growth pattern and provided a non-homogeneous Poisson process (NHPP) interpretation of the Duane model. Crow modeled the Duane postulate stochastically as an NHPP with corresponding maximum likelihood estimators (MLE) for model parameters and goodness-of-fit tests. Crow also pointed out that during reliability growth testing (RGT), failures occur as NHPP; but after that, failures happen as HPP. The Crow/AMSAA reliability growth model can be summarized as follows [35]: $E\big(N(t)\big) = \lambda t^{\beta}$ and $r(t) = \lambda\beta t^{\beta-1}$, where $N(t)$ represents the expected number of observed failures in $(0, t)$, $r(t)$ is the failure intensity or the instantaneous failures rate, λ is the scale parameter, β is the shape parameter for Crow AMSAA model $(\lambda, \beta > 0)$, and t is total reliability growth test time. When $0 < \beta < 1$, failures during development testing occur as a NHPP with a decreasing failure rate $r(t)$. For $\beta = 1$, there is no reliability growth. During development testing, $r(t)$ is decreasing because of design fixes which contribute to eliminate certain failure modes during reliability growth testing. After completion of the RGT, the inter-arrival times follow the exponential distribution and are constant, i.e., failures occur following a HPP with constant failure rate $r(T)$, and it is not cost-effective to continue reliability growth testing. The MTBF at time T and system reliability R(t) after total growth time T are: $MTBF = 1/r(T) = [\lambda\beta T^{\beta-1}]^{-1}$, and $R(t) = exp(-r(T)t) = exp\big(-\lambda\beta T^{\beta-1} t\big)$, respectively. It can be seen that the Duane model and Crow model share a similar power-law functional form to model the reliability growth with a decreasing failure rate [3].

2.2.3. Other reliability growth models literature

In addition to Duane's RGP model [33], and its extension proposed by Crow (1974) [34], RGP has been extensively investigated in the literature. Crow continued and extended the original model by introducing confidence intervals on the failure intensity and reliability functions when failure data are generated by multiple repairable systems (1990 [36] and 1993 [37]). Key parameters to reliability growth such as reliability goal setting, growth potential design margin, two failure modes, design correction effectiveness, and reliability management strategy have also been discussed [38]. Quigley and Walls (2003) [39] investigated the confidence intervals of reliability growth models and proposed a model to be applicable when the sample sizes are small. Lloyd (1986) [30] used the binomial model and introduced a model for estimating and forecasting reliability from the attribute data. Robinson and Dietrich (1987) [40] introduced a nonparametric reliability growth model, as an extension to the AMSAA model, to analyze the failure rate of a system which estimates one unknown parameter of the reliability growth model using the by the unimodal likelihood function. The method was extended to analyze the system-level reliability growth and development progress [41].

There are also some reliability growth models in literature which focus on reliability growth modeling and monitoring rather than reliability growth planning. Smith and Oren (1980) [42] proposed an effective statistic based estimator for the Duane model parameters by tabulating the number of failures between fixed points in time. The goodness-of-fit of this estimation method is compared with the Crow estimation method under varying and limited available failure sample data. Xie and Zhao (1993) [43] proposed a "first-model-validation-then-parameter-estimation" approach to simplify model validation and parameter estimation in software reliability analysis. Xie and Zhao (1996) [44] introduced alternative reliability growth models to accommodate the cases when Duane model is not well fitted. Ebrahimi (1996) [45] pointed out that one specific reliability growth model is usually valid for a certain finite amount of development time and proposed to apply a different reliability growth model to each of the design modification when the timing of such design improvements is known.

In the early product development stage, reliability growth planning becomes critical to support decision making for the overall product development program. Krasich et al. (2004) [46] compared the modified power law model with the other reliability growth models in the product design phase. They pointed out that reliability growth efforts should be shifted from the test stage to design stage to achieve a cost and time effective reliability management program. Walls and Quigley (1999 [47] and 2001 [48]) studied the product development stage reliability growth by integrating experts' engineering judgments into a mathematical reliability growth model. Coit (1998) [35] investigated how to optimally allocate limited reliability growth testing time into different subsystems to maximize the overall system reliability under testing budget constraints. The method also considers minimizing the failure rate uncertainty for each subsystem and demonstrates the significance of intelligently allocating limited testing time to subsystems with various growth potentials. Johnston et al. (2006) [49] investigated how to select reliability improvement tasks under a new system development cost and time constraints in the concept design stage. A formulated integer programming approach is used to sequence and schedule reliability improvement tasks. In addition to maximizing projected reliability, another objective, minimizing reliability estimation uncertainty, has also been considered. Heydari et al. (2014) [50] considered the allocation of testing times in reliability growth among the components of a series-parallel system with the possibility of testing at different levels, i.e., component, subsystem, and system levels. Jin et al. (2010) [51] studied a stochastic model for predicting the reliability growth for field or in-service electronic systems considering latent failure modes.

Reliability growth planning models have been investigated under the multi-phase decision process. Jin et al. (2013) [26], extended their previous work (Jin et al., 2010 [51]) in which the latent failures are incorporated into the reliability growth prediction. They developed a multi-phase reliability growth model considering the latent failures by integrating the corrective actions (CA) function into the RGT model to consider both surfaced and latent failure modes. From the product development time point of view, the focus of the work by Jin et al. (2013) [26] is on in-field products, when the product is released; however, the early design or prototyping periods, which occurs before the product release time, are not taken into consideration in their model. Jin and Li (2016) [52] proposed a lifecycle reliability growth framework in which the reliability growth efforts are integrated into design, manufacturing, and post-installation stages of a new product lifecycle. The reliability growth efforts are extended to post-installation by deploying on-site corrective actions against latent failures. Jackson (2016) [53] proposed a multi-phase reliability growth model which allows data from multiple reliability growth test phases to be aggregated in a way that provides assurance on demonstrated reliability growth, to the extent that it can be used in an acceptance testing framework. The proposed model accurately integrates reliability growth test phases based on the design dates. The incorporation of the design dates allows complex, overlapping and dissimilar phases of reliability growth testing to be accurately incorporated into subsequent analysis.

In the literature, the reliability growth planning has also been formulated as a multi-objective problem. Jin and Wang (2009) [54] formulated a reliability growth model with two objective functions including: 1) maximizes system reliability, and 2) minimizes the reliability uncertainty. The limited corrective actions budget is considered as a constraint in their bi-objective RGP model, which is solved by a genetic algorithm combined with the greedy heuristic. More recently, Awad (2016) [55] proposed a model to allocate the RG testing time which minimize system failure rate, which is modeled using Weibull distribution. The limited RG resources, e.g., time and budget, are considered in the proposed RG model. Crow (2015) [56] proposed a reliability growth projection model, called continuous evaluation reliability growth projection model, in which the planned reliability growth curve across all future multiple phases is progressively updated based on the actual data. Crow expanded the concept of the reliability growth projection at the end of a single test phase and extrapolated the reliability growth projection to the ends of multiple future test phases. The proposed model estimates the parameters for reliability growth testing over the multiple test phases.

Both multi-phase and multi-stage reliability growth models have been used in the reliability growth literature, and these two terminologies are sometimes used interchangeably. For example, in the work done by Jin et al. (2013) [26], the multi-phase reliability growth is referred to a multi-phase decision making process in terms of corrective action resource allocation for both early surfaced failure modes and subsequent latent failure modes. This multi-phase decision process repeats whenever the overall system failure prediction is updated based on design changes, retrofits, and new field failure data. Each decision-making phase includes two iterative steps: reliability prediction, and corrective action resource allocation. The multi-phase optimization model integrates the effectiveness of corrective actions, the reliability growth, and the failure prediction. The multi-phase reliability growth is also used in a reliability growth model proposed by Crow (2015) [56], which provides a reliability growth plan to continuously updating future test phase reliability projection using previous test phase data. Multi-phase in this thesis refers to the reliability growth updating over multiple test phases. More recently, the multi-phase reliability growth is investigated by Jackson (2016) [53]. The model integrates overlapping and dissimilar reliability growth curves in multiple test phases into one overall reliability growth curve. Bayesian methodology is introduced for incorporating historical reliability knowledge to estimate the MTBF using the Markov Chain Monte Carlo (MCMC) method. All these multi-phase reliability growth models realized the fact that reliability growth occurs over multiple phases or stages. However, an integrated reliability growth model to optimally allocate development budget to multiple product development stages from a product's lifecycle point of view has not been well addressed in the literature. In addition, it should be noted that the aforementioned reliability growth models are all formulated using a single objective function. The “multi-stage reliability growth” term aligning with multiple product development stages was first investigated by Li et al. (2016) [3], and then extended by Mobin et al. (2017) [57], in which multiple stages of developing a new product is considered while planning and predicting the reliability growth in the early design stages. The multi-stage reliability growth model in this research is consistent with the terminology used in Li et al. (2016) [3] and Mobin et al. (2017) [57], i.e., the multiple stages refer to concept design, detail design, and prototype design stages during the new product introduction process.

Table 2.1 summarizes the reliability models in the literature. As it is shown, most of the recently developed reliability growth planning models neither considered multi-objective in planning reliability growth, nor considered multiple stages of developing a new product. As mentioned before, there are very few studies which have considered a multi-phase decision-making process in planning the reliability growth, but they have not modeled the reliability growth through multiple stages of new product development process. This research will propose a new model for planning the reliability growth considering multiple stages as well as multiple objectives of RGP.

Table 2.1: A summary of reliability growth models literature review

Author	NPD stage	Number of objectives	Number of stages
Duane (1964) [33]	Field (test)	Single	Single
Crow (1974) [34]	Field (test)	Single	Single
Lloyd (1986) [30]	Field (test)	Single	Single
Robinson and Dietrich (1987) [40]	Field (test)	Single	Single
Coit (1998) [35]	Field (test)	Single	Single
Walls & Quigley (1999) [47]	Early design	Single	Single
Walls & Quigley (2001) [48]	Field (test)	Single	Single
Quigley and Walls (2003) [39]	Field (test)	Single	Single
Krasich et al. (2004) [46]	Early design	Single	Single
Johnston et al. (2006) [49]	Early design	Single	Single
Jin and Wang. (2009) [54]	Field (test)	Multiple	Single
Jin et al. (2013) [26]	Field (test)	Single	Single
Jin and Li (2016) [52]	Early design	Single	Single
Jackson (2016) [53]	Field (test)	Single	Single
Awad (2016) [55]	Field (test)	Single	Single
Crow (2015) [56]	Field (test)	Single	Multiple
Li et al. (2016) [3]	Early design	Multiple	Multiple
Mobin et al. (2017) [57]	Early design	Multiple	Multiple

2.3. Verification and Validation (V&V) process in early product development stages

The verification and validation (V&V) process is one of the main processes in the early stages of the NPD, which includes identifying, planning, and implementing a series of engineering activities which are proposed to meet design objectives and performance requirements, such as a desired reliability level. In general terms, verification and validation are the methods that are used for confirming that a product, service, or system meets its respective specifications and fulfils its intended purpose. The terms of “Verification” and “Validation” have been defined in literature in different ways, and have been used interchangeably, or, in some cases, are referred to “verification, validation, and testing (VV&T)” as if it were a single concept, with no apparent distinction among the three terms [2, 58]. Different definitions of verification and validation are provided in [2]. To be more specific, verification is a quality control process to assess whether a product (service or system) complies with the regulations, specifications, or conditions imposed at the early stages of development; validation is a quality assurance process of stablishing evidence that provides a high degree of assurance that a product accomplishes its intended use requirements.

The early new product design and development stages are extremely important to optimize product configuration and to verify and validate the technical, functional, regulatory, and other product performance and lifecycle requirements [59]. In addition to the product verification and validation process, the process that produces the product should also be optimally designed, and the term of processes verification and validation (V&V) is also seen during the NPD process. In this research, the focus is on product verification and validation (V&V) process, which is an integral part during the NPD process and it should be adequately deployed and planned during the early product design stages for reducing the product development cost and cycle as well as improving the reliability of the developed product.

Traditional product design and development verification and validation tools are mostly qualitative rather than being quantitative in nature. For example, Quality Function Deployment (QFD), functional decomposition and flow analysis [60, 61], Key Characteristics (KCs), Design for X (DFX) [62], and dimensional and shape verification [63] techniques have been utilized in the product development practices. These approaches have had a significant impact on improving product (process) performance in the context of the lifecycle [64, 65]. Maropouulos et al. (2010) [2] reviewed the literature of verification and validation in the context of engineering design and provided a coherent analysis and classification of V&V activities from initial design to the physical verification and validation of products and processes. In the literature, many researchers have also studied simulation and model-based design when designing a new complex engineering product/process/system. Bozzano et al. (2014) [66] proposed an automated validation approach for the spacecraft designs, which provides design checking techniques for the analysis of functional, safety, dependability, and performance requirements of the early designed system. Zentner et al. (2011) [67] proposed the application of the sensitivity analysis approach for reliable design verification of nuclear turbosets. They showed that sensitivity analysis empowers the analyst to identify the significant sources of uncertainties due to interactions of supports and sub-structures in the design, as well as, insufficient knowledge about the system by itself. Xi and Yang (2016) [68] incorporated model uncertainty into the reliability analysis during the V&V process. They investigated model parameter uncertainties using the Bayesian approach, quantified the model output uncertainties using the eigenvector dimension reduction (EDR) method, and modeled the bias calibration and approximation. The above reviewed papers addressed the product design V&V challenges focusing on product functional requirements, however, scheduling and budgeting in V&V activity planning for reliability improvement has not been well investigated in the above reviewed work.

Several studies have investigated the process of developing a new product and proposed methods to improve the process. Browning (1998) [5] explored process iteration in the NPD process and explained why some development programs do not address iteration with existing NPD project planning and control methods. Project evaluation and review technique (PERT), and its extension, the general evaluation and review technique (GERT), which are the foundational process modeling approaches and depict the design process as a progression of serial and parallel activities, also have been used in modeling the new product development process [69]. The dependency structure matrix (DSM), also known as design structure matrix (DSM), which is a model-based methodology for presenting activity relationships in an asymmetric fashion, has been applied in designing a process as well [70]. In order to analyze the distribution of expected cycle times for a process with stochastic activity duration and branching, a signal flow graph model was proposed by Eppinger et al. (1997) [71]. Ahmadi and Wang (1994) [6] modeled the iterative NPD design using the Markov chains in order to determine the composition and relation of design teams and also to estimate the NPD project cycle time. Ha and Porteus (1995) [72] considered two integrated NPD activities, including product design and production process design, and modeled the coordination between these two integrated processes by providing a trade-off between enabling greater activity overlapping and design process quality control versus requiring review and preparation time. Ford et al. (1998) [73] proposed a system dynamic-based model for the NPD process which allows for the iteration of activities and deals with activities and processes. Their model describes the dynamic structure for modeling integrated development processes separately from project resources, scope, and targets. All the above studies have been focusing on the NPD process scheduling, activity iteration, and cost issues when developing a new product and have not considered V&V activity planning for reliability improvement.

One of the very few studies in the literature that considers both product reliability and product development cycle and cost is the model proposed by Ahmed and Chateauneuf (2014) [74], in which they investigated the reliability validation of engineering products at early design stages in order to optimize the number of test units and the costs of the product design. The proposed model couples the design and testing problems and provides the optimal number of tests to meet the reliability goal considering the design, failure, and validation costs. However, their model lacks the scheduling of tests implementation and the sequencing of conducting tests.

From the perspective of improving the reliability of a new product, the verification and validation process in the early stages aims to plan and implement a sequence of identified reliability risk mitigation activities for product reliability improvement. These V&V activities can be obtained from the design failure mode and effect analysis (DFMEA) process, and have to be executed on schedule and under the budget constraint by the new product release time. Although the reviewed models partially captured some characteristics of implementing the V&V activities in a NPD process, such as implementation budget and schedule, iteration of some processes, changes in activity content with iterations, and also the relationship between product design and process design activities, most of them fall short of quantitatively modeling the V&V activities planning process by comprehensively including and considering the time, cost and reliability of a new product. In addition, most existing product design V&V planning processes are based on qualitative methods, and there is very limited research that has studied quantitative methodologies for modeling and optimizing the product design V&V activities by considering the effectiveness of each activity and the sequencing constraint of implementing multiple activities. In this research, a quantitative approach is developed to plan the product design V&V activities which considers the cost, time, and effectiveness of V&V activities as well as the reliability improvement of the product after implementing the V&V activities.

2.4. Summary

The existence of tremendous pressure on today's companies to achieve and sustain competitive advantages has led to large efforts to reduce the product development cycle time and cost while providing reliable products. There have been many studies to model and optimize the new product development process via a reliability growth program, but most of them are focused on the later stages of the NPD process when the product test data become available, and a few research has considered the reliability of the product during the early stages of developing a new product. However, in the early product development stages, such as during the concept design stage and product portfolio selection stage, actual testing data are rarely available for RGP modeling. In addition, most existing RGP models consider a single product development stage by optimizing the single reliability maximization objective. But the reliability improvement process usually goes through multiple stages according to the multiple stages of developing a new product. Furthermore, the time and cost associated with the RGP should be considered when the reliability growth program is being planned, but the existing RGP models have not considered all these objectives, i.e., time, cost, and product reliability, simultaneously.

In addition to the lack of research in the literature about the reliability growth planning during the early stages of new product development considering multiple objectives of the new product development process, there are few research studies that optimally plan the design verification and validation activities, which is one of the major processes of achieving product desired reliability. V&V activities should be implemented in predefined sequences, under budget, and on schedule, in order to mitigate the defined failure modes of the product under development. There is no mathematical approach to model and optimize the V&V planning in the literature which considers all multiple objectives of the NPD process.

In this research, two major reliability improvement activities during development process of a new product are modeled and optimized. These two reliability improvement activities include: 1) reliability growth planning (RGP); and 2) product design verification and validation (V&V) activities planning. First, a model for a multiple-stage multiple-objective reliability growth planning of a new product development process is investigated. A new optimization approach is provided to solve the multiple-stage multiple-objective reliability growth planning model. In the second part of this research, a new quantitative approach is studied which mathematically models the product design verification and validation activities planning.

Chapter 3. Multi-Objective Multi-Stage Reliability Growth Planning (MO-MS-RGP)

3.1. Introduction

In this chapter, a multi-objective multi-stage reliability growth planning method in the early product development stage is investigated. Multi-stage reliability growth planning is common in practice and it aligns well with multiple developmental stages of a new product such as concept design, detail design, prototype design, and final production version design. The multi-objective formulation reflects the needs of product development's multiple objectives, e.g., program cost, schedule, and reliability. An improved non-dominated sorting genetic algorithm (NSGA-II) is presented to solve a multi-objective multi-stage formulation for reliability growth planning. Data envelopment analysis (DEA) methods are introduced to reduce the Pareto optimal solutions to a workable size of efficient solutions for plan implementation. An illustrative example is given to demonstrate the approach for planning the reliability growth for a next generation engine development. The integrated optimization algorithm consists of NSGA-II and DEA tools are applied to optimize the mathematical model. Through trade-off and sensitivity analysis, insights and guidelines are provided for choosing appropriate reliability growth plans in terms of optimal allocation of testing time, testing units and timing for new technology introduction. Other factors such as growth rates which influence product development objectives including development cost, schedule, and reliability target, are also discussed in this chapter.

3.2. Reliability growth planning

As illustrated in Figure 3.1, which represents one example of a new product development plan, a new product, especially a modern complex engineering system, usually goes through multiple developmental stages such as concept design, detailed design, and production design stages. New technologies, which are represented as new subsystems in Figure 3.1, can be introduced to the product design concurrently or sequentially. While developing a new system, its reliability is usually very challenging to estimate in each stage due to the unique design contents and characteristics in each stage. To achieve more accurate and realistic model for reliability growth planning, reliability growth needs to be planned in multiple stages in order to align with the multiple product developmental stages.

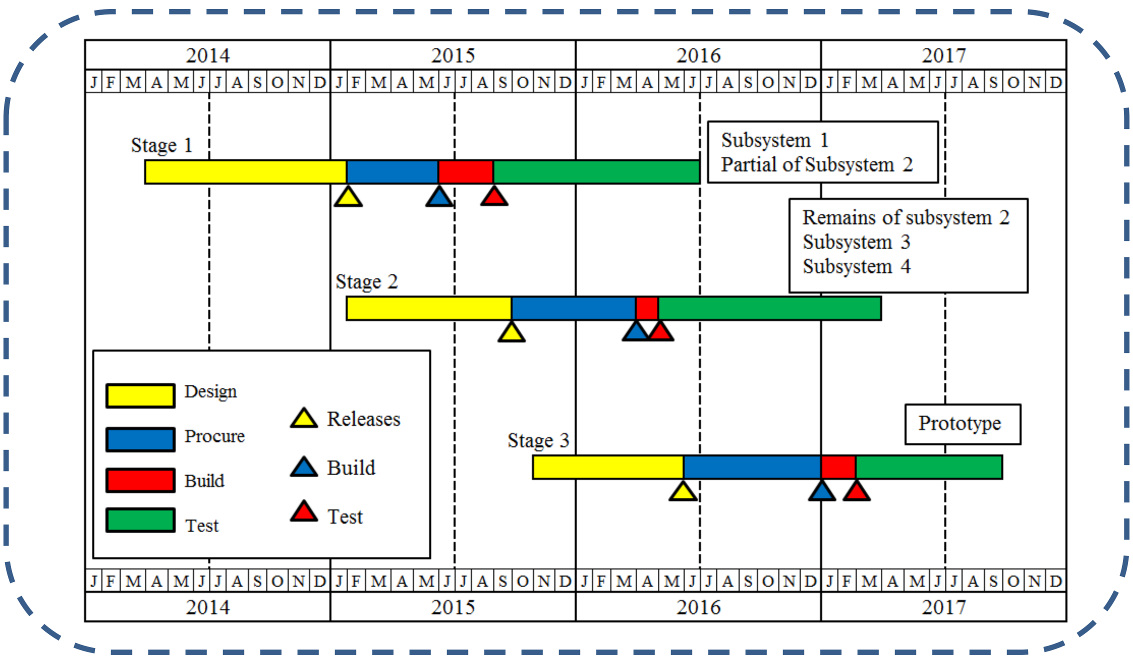

Figure 3.1: An example of multi-stage New Product Development (NPD) plan

An example of single stage reliability growth plan is presented in Figure 3.2. For a single reliability growth planning, typical concerns and key parameters include how to estimate the initial system reliability in terms of mean time between failure (MTBF), how to set the reliability growth starting time, how to determine an appropriate growth rate for a specific product design considering its configuration, and how to optimize the reliability of the system considering limited recourses such as test units and time [42, 75].

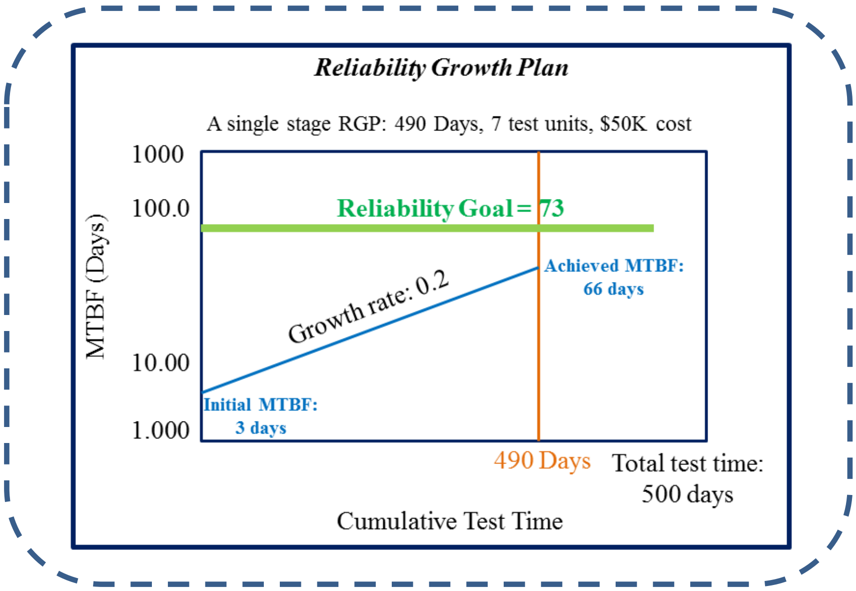

Figure 3.2: An example of a single stage reliability growth plan

For multiple-stage reliability growth planning, other challenges exist to obtain the optimal reliability growth planning. The schematic in Figure 3.3 shows one typical reliability growth planning outcome by considering the key planning factors for a three stage product development process. The challenges in modeling the multiple stage reliability growth planning include:

(1) Since new contents (systems/modules involve new technologies), with different mechanical and electrical configurations, will be added to the system during multiple stages, it is challenging to quantify how the residual new contents from previous stages can be carried over to the next stage reliability growth. Correlating the obtained reliability growth at the end of each stage, to the initial reliability estimation of the product at the next stage is the key issue.

(2) Considering the limited test budget and time during a reliability growth program, it is important to optimally allocate test units and time to individual stage;

(3) Considering that new technologies can be added to the system, as a new sub-system, concurrently and/or sequentially, it is important to find out that when and what percentage of new contents/technologies should be introduced at each stage.

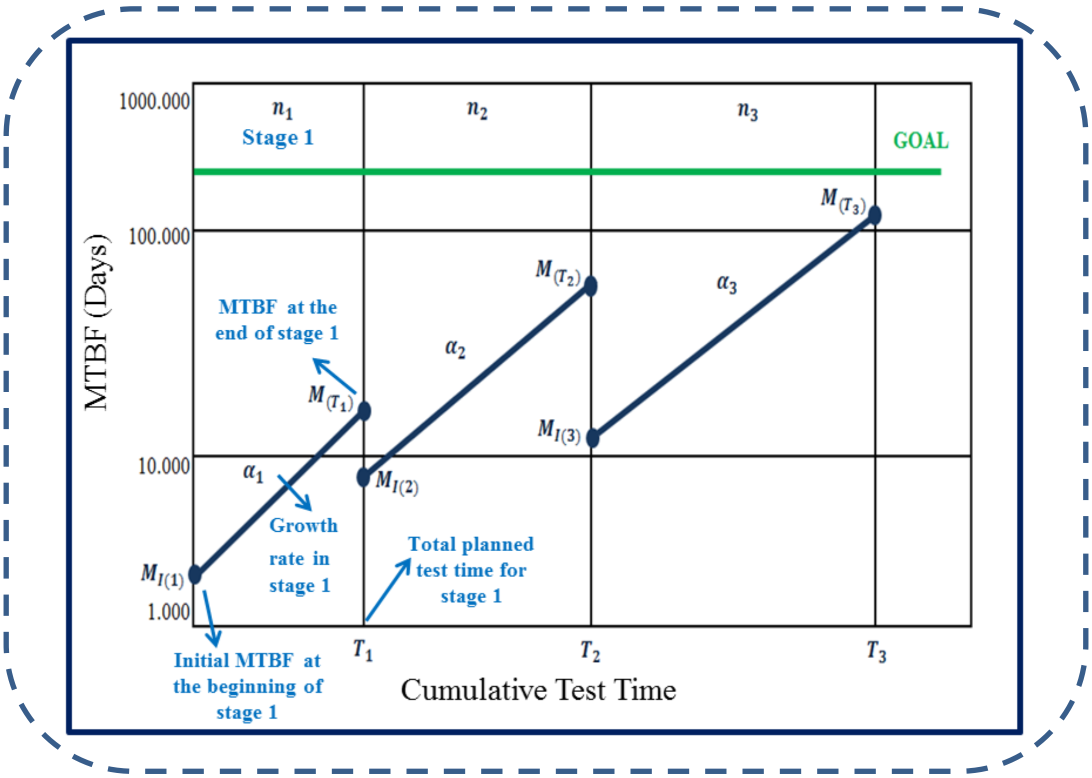

Figure 3.3: The schematic of multi-stage reliability growth planning

These challenges need to be addressed during the new product development process. More specifically, the following major elements need to be determined for a multi-stage reliability growth planning:

(1) Reliability growth rate (α_i), which is usually empirically determined and can vary from stage to stage during a new product's development process should be defined. The characteristics of the system, such as mechanical and electrical configuration of the system, and the effectiveness of reliability program are major factors in determining the reliability growth rates. Historical data for reliability growth from previous product development experiences can also be used as references.

(2) New technologies which are added to the system as new contents also should be analyzed for accurate reliability growth planning. New technology introduction and timing will have a significant impact on reliability projection. Reliability growth estimation depends on the amount and time of introduced new technologies. In the process of developing a new product, a new technology can be introduced in different stages. The new contents' information in the bill of materials (BOM) which includes the components of the product and their configurations, and also the failure rates estimation can be used for initial failure rate estimations of the developing product in each stage.

In this research, a reliability growth planning model is developed which integrates the multiple stages of developing a new product process. The multi-stage reliability growth planning model consists of multiple objectives including the product reliability in terms of failure rate, new product development time, and total development cost. Since the reliability growth process always encounters time and cost constraints, boundary values will be placed on the testing time and the number of test units. A recently developed version of a multi-objective evolutionary algorithm, which has been proven to be effective in solving multi-objective optimization problems in literature, is modified and applied to arrive at the optimum reliability growth plans. After obtaining a set of optimum reliability growth plans in the format of Pareto frontier, and in order to reduce the number of optimum reliability growth plans into a workable size, the optimum solutions are compared in terms of relative efficiency using the data envelopment analysis approach. The result of applying the data envelopment analysis (DEA) approach is a set of efficient optimal reliability growth plans that can be implemented during the multiple stages of developing a new product.

The presented modeling and optimization approaches in developing a new product is applied to the case study of the next generation of engines. In order to provide new product developers with more insights about the reliability growth planning, a trade-off analysis between new product reliability and new product development cost and time, as well as a sensitivity analysis of RGP model's parameters will be provided.

In summary, a list of research tasks to achieve the first objective of this research is presented as follows:

- Providing a mathematical model for the multi-objective and multi-stage reliability growth planning.
- Developing an integrated algorithm for the multi-objective optimization of reliability growth planning, including fast non-dominated sorting genetic algorithm (NSGA-II) to generate optimum solutions, and data envelopment analysis (DEA) to reduce the number of optimal solutions.
- Conducting a trade-off analysis between product reliability and product development cost and time.

3.3. Multi-Objective and Multi-Stage Reliability Growth Planning (MO-MS-RGP)

In this section, the mathematical modeling of the multi-stage multi-objective reliability growth planning is presented. The notations that are used in formulating the multi-objective multi-stage reliability growth planning are presented as follows:

j	Index of the added subsystem in each stage ($j = 1, 2, \dots, m_i$)
i	Index for the stages of developing a new system ($i = 1, 2, \dots, n$)
m_i	Number of subsystem in stage i
n	Number of stage ($i = 1, 2, \dots, n$)
α_i	The growth rate in stage i
n_{ij}	Number of test units for subsystem j in stage i
$n_{l_{(ij)}}$	Lower bound on the number of test units for subsystem j in stage i
$n_{u_{(ij)}}$	Upper bound on the number of test units for subsystem j in stage i
N_i	Total number of test units in stage i
$N_{l(i)}$	Lower bound for the number of test units in stage i
$N_{u(i)}$	Upper bound for the number of test units in stage i
N	Total number of test units
t_{ij}	Planned testing time for each subsystem j in stage i
T_i	Total planned test time for stage i
T	Total planned test time for all stages
τ_u	Upper bound of total test time for reliability growth test
τ_i	Development time for stage i, which is the maximum planned testing time in stage i, i.e., $\tau_i = \max\{t_{ij}\}$, for all j
$t_{l_{(ij)}}$	Lower bound on the test time for subsystem j in stage i
$t_{u_{(ij)}}$	Upper bound on the test time for subsystem j in stage i
τ	Total development time, $\tau = \text{sum}\{\tau_i\}$
c_{ij}	Fixed cost of subsystem j in stage i
$C_{v(i)}$	Variable cost which is a function of total test time for stage i (T_i)

C_i	Total test cost of stage i, including fixed cost and variable cost
C	Total test cost
λ_{ij}	Failure rate of subsystem j in stage i
θ_i	Percentage of introduced new contents in stage i, ($\sum_{i=1}^{n} \theta_i = 100\%$)
$\lambda_{n(i)}$	Failure rate of all new subsystems in stage i
$\lambda_{I(i)}$	Initial failure rate in stage i, which is the summation of new contents $\lambda_{n(i)}$ and residual failure rate from previous stage λ_{i-1}
W	The effective working hours in one year
V	The unit variable cost for each hour of testing
λ_i	Total failure rate at the end of stage i
$M_{I(i)}$	Initial MTBF in stage i
$M(T_i)$	MTBF at the end of stage i

It has to be mentioned that in the multi-objective and multi-stage reliability growth planning model, it is assumed that the reliability growth follows the basic Duane and Crow models assumptions, i.e., the failure rate is linearly decreasing over cumulative testing time or the mean time to failure is linearly increasing over cumulative testing time on a log-log scale. In the presented model, the failure rate at the end of stage i (λ_i) is a function of initial failure rate of the stage ($\lambda_{I(i)}$), growth rate (α_i) and the total planned testing time in each stage (T_i). The initial failure rate of each stage ($\lambda_{I(i)}$) is the summation of new contents' failure rate ($\lambda_{n(i)}$) and the residual failure rates from a previous stage (λ_{i-1}). The parameter θ_i in the model represents the proportion of introduced new contents for each stage, and it is also used to determine the development cost of subsystem j in stage i.

Considering the assumptions and notations, the multi-objective and multi-stage reliability growth planning (RGP) is formulated as follows:

$$Min:\ \lambda_{i=n} = f\left(\lambda_{\ i-1}, \lambda_{\ n(i)}, \alpha_i\,, T_i\ \right) \tag{3.1}$$

$$Min:\ \ \tau = \ \sum_{i=1}^{n} \tau_i\ ,\ \ i = 1, \dots, n \tag{3.2}$$

$$Min:\ \ \ C = \ \sum_{i=1}^{n} C_i\ \ ,\ i = 1, \dots, n \tag{3.3}$$

s.t.

$$0 \le \tau \le \tau_u \tag{3.4}$$

$$N_{l(i)} \le N_i \le N_{u(i)} \tag{3.5}$$

$$t_{l_{(ij)}} \le t_{ij} \le \ t_{u_{(ij)}}, i = 1, 2, \dots, n\ and\ j = 1, 2, \dots, m_i \tag{3.6}$$

$$n_{l_{(ij)}} \le n_{ij} \le \ n_{(ij)}, i = 1, 2, \dots, n\ and\ j = 1, 2, \dots, m_i \tag{3.7}$$

where:

$$\tau_i\ = \max_j\{t_{ij}\}\ ,\ j = 1, \dots, m_i \tag{3.8}$$

$$\tau = \ \sum_{i=1}^{n} \tau_i \tag{3.9}$$

$$T_i\ = \ \sum_{j=1}^{m_i} t_{ij}\, n_{ij} \tag{3.10}$$

$$T = \ \sum_{i=1}^{n} T_i \tag{3.11}$$

$$C_i = \ \sum_{j=1}^{m_i} (\theta_i c_{ij}\, n_{ij}) + C_{v(i)} \tag{3.12}$$

$$C_{v(i)}\ \ = \ f(\tau_i\,) \tag{3.13}$$

$$C = \ \sum_{i=1}^{n} C_i \tag{3.14}$$

$$N_i = \ \sum_{j=1}^{m_i} n_{ij} \tag{3.15}$$

$$N = \ \sum_{i=1}^{n} N_i \tag{3.16}$$

$$ln\,\lambda_i\ = ln\,\lambda_{\ I(i)} - \ \alpha_i\, ln\,T_i \tag{3.17}$$

$$\lambda_{I(i)} = \lambda_{\ n(i)} + \ \lambda_{\ i-1}\ (i = 1, \dots, n) \tag{3.18}$$

$$\lambda_{\ \mathrm{n(i)}} = \sum_{\mathrm{j=1}}^{\mathrm{m_i}} \lambda_{\mathrm{ij}}\ \ \ (\mathrm{j} = 1, \dots, \mathrm{m_i}) \tag{3.19}$$

$$M_{I(i)} = \ \frac{1}{\lambda_{I(i)}}\ \ (\ i = 1, \dots, n) \tag{3.20}$$

$$ln[M(T_i)]\ = ln[M_{I(i)}] + \ \alpha_i * ln[T_i]\ \ \text{or}\ \ ln\,\lambda_i\ = ln\,\lambda_{\ I(i)} - \ \alpha_i\ ln\,T_i \tag{3.21}$$

There are two sets of decision variables in this model: 1) the number of test units for subsystem j in stage i, denoted as n_{ij}; and 2) the planned testing time for each subsystem j in stage i, denoted as t_{ij}. There are three objectives in this optimization problem. The first objective is to minimize the failure rate at the last stage (Equation 3.1), when $i = n$, which is the function of the residual failure rate from previous stage (λ_{i-1}), new contents or introduced technologies in current stage ($\lambda_{n(i)}$), reliability growth rate (α_i) and planned testing time for the stage (T_i) (Equation 10). The second objective (Equation 3.2) is to minimize the total development time (τ) which is the summation of the development time for all stages (τ_i). The last objective is to minimize the total test cost (C) (Equation 3.3) which is the summation of all fixed and variable costs of all stages (Equation 3.10). There are two constraints in this optimization problem which are the constraint on total product development and testing time (Equation 3.4) and lower and upper bounds for the number of available test units in each development stage (Equation 3.5). There are also several sets of constraints in the MO-MS-RGP problem which includes: upper bound on total product development and testing time (Equation 3.4); lower and upper bounds for the number of available test units in each development stage (Equation 3.5); the lower and upper bounds on the test time of each subsystem in each stage (Equation 3.6); the lower and upper bounds on the number of test units for each subsystem in each stage (Equation 3.7),

3.4. Solution methodology

3.4.1. Multi-objective optimization algorithm

There are numerous multi-objective optimization techniques modified and applied in the literature for various optimization problems. In general, there are two approaches for solving multiple objective optimization problems [82]. The first approach is to integrate all objective functions as a single composite function. The second method is to select one of objectives as the main objective and consider others as constraints. Both methods have some difficulties in practice [76, 77]. For example, determining an appropriate utility function such as selecting appropriate weights for each objective becomes very difficult when combining multiple incomparable objectives [78, 79]. Considering some objectives as constraints by specifying boundary values may reduce the solution space and lose candidate solutions preferable to decision makers [80, 81].

One meta-heuristic method which is especially appropriate for multi-objective problems is genetic algorithm [82]. Holland developed the traditional genetic algorithm (GA), which is particular class of evolutionary algorithms [83]. Genetic algorithms start with a population of random individuals which are called chromosomes. Each chromosome represents a unique solution in the solution space. Crossover and mutation are common approaches for generating new solutions. Solutions are evaluated in terms of a fitness function which determines the probability of solutions' survival for the next generation. The algorithm continues for a pre-determined number of generations or until no additional improvement is observed.

To solve a multi-objective optimization problem, significant research has been carried out to develop multi-objective heuristic algorithms. Specially, non-dominated sorting GA or NSGA, developed by Srinivas and Deb [84], is a popular non-domination based GA which uses a non-dominated sorting procedure and applies a ranking method that emphasizes those good solutions and tries to maintain them in the population. A solution is said to dominate another if the objective functions of it is no worse than the other and at least in one of its objective functions it is better than the other. In addition to the non-dominated sorting approach which sorts the solutions considering all objective functions, the NSGA algorithm maintains the diversity in the population through the sharing method. The algorithm explores different regions in the Pareto front and is very efficient in obtaining sufficient Pareto optimal sets. Although NSGA is a very effective algorithm, it has been generally criticized for its computational complexity, lack of elitism and for choosing the optimal parameter value for sharing parameter. A modified version of NSGA, NSGA-

II, developed by Deb et al. [85] utilizes a fast non-dominated sorting genetic algorithm. This method is more computationally efficient, non-elitism preventing, and less dependent on the sharing parameter for diversity preservation. In this research, the MO-MS-RGP is solved using NSGA-II with moderate modification and adaption to effectively represent the solutions and search the Pareto front. The specific aspects of NSGA-II are described as follows:

1- A random parent population (P_0) is initialized and sorted by using the fast non-dominated sorting algorithm into each front (F_i) in order to identify the non-dominated fronts.

2- The first front (F_1) is a completely non-dominant set in the current population and the second front (F_2) is dominated by the solutions in the first front only and the front iterates in the same manner.

3- Each solution in each front is assigned a rank (fitness value) or front to which they belong.

4- In addition, fitness value, a new parameter called crowding distance is calculated for solution in each front (F_i) which is the measure of population density around a solution and helps to obtain a uniform spread of solutions along the best-known Pareto front without using a fitness sharing parameter [82]. The basic idea behind the crowding distance is finding the Euclidian distance between each solution in each front. The solutions in the boundary are always selected since they have infinite distance assignment. Large average crowding distance will result in better diversity in the population.

5- Parents are selected from the population by using binary tournament selection based on the rank and crowding distance. If the solutions are in the same non-dominated front, the solution with higher crowding distance is selected. Otherwise, the solution with the lowest rank is selected. In this step, the simulated binary crossover (SBX) [3] and polynomial mutation operators [3] are applied in order to generate offspring from the selected population.

6- Based on the selection process, the individuals of next generation are selected from the current population and current offspring. As long as all the previous and current best solutions (or elitist solutions) are maintained in the next population, elitism is ensured. This population, which includes both current population and current offspring, is sorted again based on non-domination and only the best N solutions are selected, where N is the population size [82].

3.4.2. Pareto optimal solution reduction using data envelopment analysis (DEA)

Even though the results of NSGA-II are informative and can provide trade-off information for the multiple objectives, the number of solutions may still be prohibitive for a decision maker to make choices [86, 87]. At this point, selecting representative solutions from all our solutions obtained from NSGA-II itself can be considered as a multi-objective optimization problem, also called multiple objective selection optimizations (MOSO) problem [88]. In fact, the appropriate application of an MOSO method can significantly reduce the size of solutions from Pareto optimal solutions [57, 89, 90]. A special MOSO method is the data envelopment analysis (DEA) method. From the perspective of relative efficiency, DEA is able to eliminate those inefficient Pareto optimal solutions. In the context of the design of multi-objective reliability growth plans, those plans with high reliability and lower testing cost and time are preferred and need to be selected for plan implementation.

Data Envelopment Analysis (DEA) which was originally introduced by Charnes et al. [91] is a technique for measuring the relative performance of decision making units (DMUs). This method is based on linear programming methods and it addresses the difficulties of comparing DMUs which use multiple inputs (i.e., cost type criteria) to produce multiple outputs (i.e., benefit type criteria) [92]. For MOSO, each alternative solution is considered as a DMU in the DEA method, and all the DMUs are usually assumed to be homogeneously comparable such that the resulting relative efficiencies are meaningful. In comparing their efficiencies, the relative efficiency incorporating multiple inputs and outputs can be defined [91, 90].

In DEA, a ratio of a weighted sum of outputs to a weighted sum of inputs is calculated as a measure of efficiency of each DMU. Consider a set of n DMUs, with each DMU j, $(j = 1, \ldots, n)$ using m inputs x_{ij} $(i = 1, \ldots, m)$ and generating s outputs $y_{rj}(r = 1, \ldots, s)$. If the weights (price or multipliers) $\bar{u}_r$ and $\bar{v}_i$ associated with output r and input i, respectively, are known, the efficiency $(\bar{e}_j)$ of DMU_j as the ratio of weighted outputs to weighted inputs is equal to $\sum_r u_r y_{rj} / \sum_i v_i x_{ij}$. In 1987, Charnes et al. [91] proposed their DEA model (known as CCR) which is the constant returns to scale (CRS) model in the absence of known multipliers. Their model measures the technical efficiency of DMU_0 by solving the following fractional programming problem, known as original CCR input-oriented model:

$$\begin{aligned} & e_0 = max \textstyle\sum_r u_r \, y_{r0} \, / \textstyle\sum_i v_i x_{i0} \\ \text{s.t.} \quad & \textstyle\sum_r u_r y_{rj} - \textstyle\sum_i v_i x_{ij} \le 0, all\ j \\ & u_r, v_i \ge \varepsilon \, , \forall \, r, i. \end{aligned} \tag{3.22}$$

where ε is a non-Archimedean element smaller than any positive real number. Since this model is involving the ratio of outputs to inputs, it is referred to as the input-oriented model. The output oriented model is the inverted form of this ratio with minimization objective.

By making the change of variables $\mu_r = tu_r$, and $v_i = tv_i$, where $t = (\sum_i v_i x_{i0})^{-1}$, the previous fractional programming problem can be converted to linear programming (LP) model, known as the envelopment or primal problem:

$$
\begin{aligned}
& e_0 = max \sum_r \mu_r y_{r0} \\
\text{s.t.} \quad & \sum_i v_i x_{i0} = 1 \\
& \sum_r \mu_r y_{rj} - \sum_i v_i x_{ij} \le 0, \ \forall j \\
& \mu_r , v_i \ \ge \ \varepsilon, \ \forall \, r, i.
\end{aligned}
\tag{3.23}
$$

The duality of the previous problem is a linear programming problem known as the multiplier or dual problem which provides detailed information for relative efficiency measure.

$$
\begin{aligned}
& min \, \theta_0 - \varepsilon \left(\sum_r S_r^+ + \sum_i S_i^-\right) \\
\text{s.t.} \quad & \sum_j \lambda_j x_{ij} + S_i^- = \theta_0 x_{i0} \, , i = 1, \dots, m \\
& \sum_j \lambda_j y_{rj} + S_r^+ = y_{r0} \, , r = 1, \dots, s \\
& \lambda_j, S_i^-, S_r^+ \ge 0 \, , \forall i, j, r \\
& \theta_0 \ unconstrained
\end{aligned}
\tag{3.24}
$$

where S_i^- and S_r^+ are slack variables [3, 89].

The other DEA model is introduced by Banker et al. [93], i.e., the BCC model, which is the extension of CCR model and is fundamentally the variable returns to scale (VRS) model. The BCC model differs from CCR model, by way of an additional variable (u_0).

The linear programming of the BBC model is:

$$
\begin{aligned}
& e_0^* = \max \textstyle\sum_r \mu_r y_{r0} - \mu_0 \\
& \text{s.t.} \quad \textstyle\sum_i v_i x_{i0} = 1 \\
& \qquad \textstyle\sum_r \mu_r y_{rj} - \mu_0 - \sum_i v_i x_{ij} \le 0, \;\; j = 1, \dots, n \\
& \qquad \mu_r, v_i \ge \varepsilon, \forall\, r, i. \qquad \mu_0 \; unrestricted \; in \; sign.
\end{aligned}
\tag{3.25}
$$

The dual of this BCC model is:

$$
\begin{aligned}
& \min \theta_0 - \varepsilon \left(\textstyle\sum_i s_i^- + \sum_r s_r^+\right) \\
& \text{s.t.} \quad \textstyle\sum_j \lambda_j x_{ij} + S_i^- = \theta_0 x_{i0}, \;\; i = 1, \dots, m \\
& \qquad \textstyle\sum_j \lambda_j y_{r0} - S_r^+ = y_{r0}, \;\; r = 1, \dots, s \\
& \qquad \textstyle\sum_j \lambda_j = 1 \\
& \qquad \lambda_j, S_i^-, S_r^+ \ge 0 \;\; \forall i, r, j \qquad \theta_0 \; unrestricted \; in \; sign.
\end{aligned}
\tag{3.26}
$$

The dual of BCC differs from the dual of CCR in a way it has the additional convexity constraints on the λ_j ($\sum_j \lambda_j = 1$). In both CCR and BCC model, the performance of $DMU_0$is fully (100%) efficient if and only if both (i) $\theta_0^* = 1$ and (ii) all slacks $S_i^{-*} = S_r^{+*} = 0$ and weakly efficient if and only if both (i) $\theta_0^* = 1$ and (ii) $S_i^{-*} \neq 0$ and/or $S_r^{+*} \neq 0$ for some i and r in some alternative optima. Clearly, any CCR-efficient DMU is also BCC-efficient, but BCC-efficient solutions may not be CCR-efficient. Thus, we would expect more efficient solutions from BCC model and fewer efficient solutions from CCR model. The CCR model is referred to as giving a radial projection. Particularly, each input is reduced by the same proportionality factor θ. The BCC model provides more flexible projection by providing decreasing, increasing and constant return to scale frontier [3, 89]. In order to guarantee that optimal solutions are efficient, all theses four DEA approaches are used to identify the efficient optimal solutions, and solutions that are identified as efficient using four DEA approaches are introduced as efficient optimal solutions.

3.5. Case study: Application of MO-MS-RGP in developing next generation dual fuel engines

In the early product development stage, strategic decisions such as the amount of new technology introduction and product portfolio selection usually need to be made based on projected program budget, schedule, and reliability of the new product. Accurate and realistic reliability growth planning under limited available product information is a challenge but very beneficial for product development program-level decision making.

In this section we demonstrate how the multi-objective and multi-stage reliability growth planning method can be applied to the case of developing a next generation diesel-gas duel fuel turbine engine. Diesel engine was invented over one hundred years ago; however, the advancement and progress to develop new generation diesel engines have never been slowed down. These new engine developments can be attributed to factors including the continuously increasing standard of emission requirements from the US Environmental Protection Agency (EPA), expected higher mission reliability from customers, and technology advancements of electrical engine control systems. For example, the Tier-4 emissions standard from EPA will be enforced for all newly manufactured heavy duty locomotive engines in 2017. Recently, due to the prediction of abundant natural gas reserve in the USA as well as predicted cheaper price of liquid natural gas (LNG) than that of diesel fuel, a few companies including Caterpillar and General Electrics are developing the new diesel-gas duel fuel engines. These new developments and requirements bring many challenges to maintain high reliability performance under more stringent emission requirements and the introduction of advanced engine control systems.

3.5.1. MO-MS-RGP for next generation engine development

Based on the overall product development schedule, there are three major development stages for the next generation engine, concept engine, the prototype engine, and the pilot engine development. From previous product development experiences and historical reliability data, growth rates for each development stages are 0.4, 0.3, and 0.2, respectively. It is common to apply relatively higher growth rates at the earlier product development stages than those at the latter product development stages. For example, the prototype development is conducted in-house and discovered failure modes can be more effectively resolved, while as the pilot product is tested and verified at customers' facilities, failure modes may not be resolved as effectively as in-house developments. In practice, new contents/technologies are introduced in different stages of new product development. In this case, θ_i represents the percentage of introduced new contents in each stage, which also affects the amount of fixed cost of new contents introduction. In each stage, different number of subsystems (m_i) with different failure rate (λ_{ij}) for each subsystem are tested for reliability growth purpose. To avoid cost overrun in product development, we impose an upper bound for the number of testing units for each subsystem. Table 3.1 shows the parameters for multi-objective multi-stage RGP problem formulation. The failure rate value for each subsystem in each stage (λ_{ij}) represents the failure rate of the introduced contents (defined as θ_i% of the entire subsystem) added to the existing subsystem which added to the system in earlier stages. For example, 80% of fuel system which added to the system in stage 2 has the failure rate of 0.20 and 20% of it which added to system in stage 3 has 0.20 failure rate.

Table 3.1: Parameters of the case study for multi-objective and multi-stage RGP

i stage	Group description	λ_{ij}	θ_1 (%)	θ_2 (%)	θ_3 (%)	c_{ij} ($1000)	m_i	α_i	$C_{v(i)}$ ($1000)	$N_{l(i)}$	$N_{u(i)}$
1	Engine Block	1.38	100	0	0	600	2	*0.4*	$2*(T_1$*2000)	4	8
	Turbocharger	0.03	100	0	0	45					
2	Engine control	0.24	0	80	0	50	4	0.3	$4*(T_2$* 2000)	8	16
	Cooling System	0.02	0	100	0	30					
	Fuel System	0.20	0	80	0	40					
	Lubricating system	0.05	0	80	0	20					
3	Engine control	0.24	0	0	20	12.5	3	0.2	$3*(T_3$*2000)	6	20
	Fuel system	0.20	0	0	20	10					
	Lubricating system	0.05	0	0	20	5					

To avoid cost overrun in product development and also since the number of test units are limited for each subsystem in each stage, an upper bound for the number of testing units for each subsystem j in stage i is imposed. In addition, the upper bound is considered for the total product development time (τ_u: 3.5 years). The variable test cost is assumed to be $2000 per hour. The variable cost ($C_{v(i)}$) is the product of T_i and the unit variable cost for each hour of testing (which is considered to be $2000 in this case study, $V = 2000$). It is also considered that there are 2000 effective working hours in each year ($W = 2000$). The RGP optimization formulation for this case study is as follow:

$$\textit{Min: } \lambda_{i=3} = f\left(\lambda_{\,I(i-1)}, \lambda_{\,n(i)}, \alpha_i\,, T_i\,\right) \tag{3.27}$$

$$\textit{Min: } \sum_{i=1}^{n} C_i = C_1 + C_2 + C_3 = (60 * n_{11} + 45 * n_{12}) + 2*(\tau_1 * 2000) + (50 * n_{21} + 30 * n_{22} + 40 * n_{23} + 20 * n_{24}) + 2*(\tau_2 * 2000) + (12.5 * n_{31} + 10 * n_{32} + 5 * n_{33}) + 2*(\tau_3 * 2000) \tag{3.28}$$

$$\textit{Min: } \tau = \sum_{i=1}^{n} \tau_i = \tau_1 + \tau_2 + \tau_3 \tag{3.29}$$

s.t.

$$0 \le \tau \le \tau_u \Longrightarrow 0 \le \tau_1 + \tau_2 + \tau_3 \le 3.5 \tag{3.30}$$

$$N_{l(1)} \le N_1 \le N_{u(1)} \implies 4 \le n_{11} + n_{12} \le 8 \tag{3.31}$$

$$N_{l(2)} \le N_2 \le N_{u(2)} \implies 8 \le n_{21} + n_{22} + n_{23} + n_{24} \le 16 \tag{3.32}$$

$$N_{l(3)} \leq N_3 \leq N_{u(3)} \Longrightarrow 6 \leq n_{31} + n_{32} + n_{33} \leq 20 \tag{3.33}$$

There are 18 decision variables in this MO-MS-RGP formulation including the number of test units for subsystem j in stage i (n_{ij}) and the planned testing time for each subsystem j in stage i (t_{ij}). The constraints related to the decision variables and the defined interval for each are presented in Table 3.2.

Table 3.2: Decision variables of the Mo-MS-RGP case study

n_{ij}	[min max]	t_{ij}	[min max]
n_{11}	[2 4]	t_{11}	[0 0.5]
n_{12}	[2 4]	t_{12}	[0 0.5]
n_{21}	[4 8]	t_{21}	[0 1.5]
n_{22}	[4 8]	t_{22}	[0 1.5]
n_{23}	[4 8]	t_{23}	[0 1.5]
n_{24}	[4 8]	t_{24}	[0 1.5]
n_{31}	[3 15]	t_{31}	[0 1.5]
n_{32}	[3 15]	t_{32}	[0 1.5]
n_{33}	[3 15]	t_{33}	[0 1.5]

The NSGA-II algorithm is used to solve the problem. The algorithm is modified to deal with both continuous (t_{ij}) and discrete decision (n_{ij}) variables. The number of population and generations in each population are set to be 100 and 40 respectively. Therefore, 100 solutions are found in the Pareto optimal set. The results of solving this RGP optimization problem are presented in the next section.

3.5.2. RGP Pareto solutions and trade-off analysis

The NSGA-II algorithm is applied to solve the RGP problem with appropriate setting for parameters such as the probability of mutation and crossover, and termination conditions. Multiple runs of the NSGA-II algorithm to the RGP formulation generate a very stable Pareto optimal frontier as shown in Figure 3.4, which represents 100 Pareto optimal solutions with given growth rates and new contents for each stage.

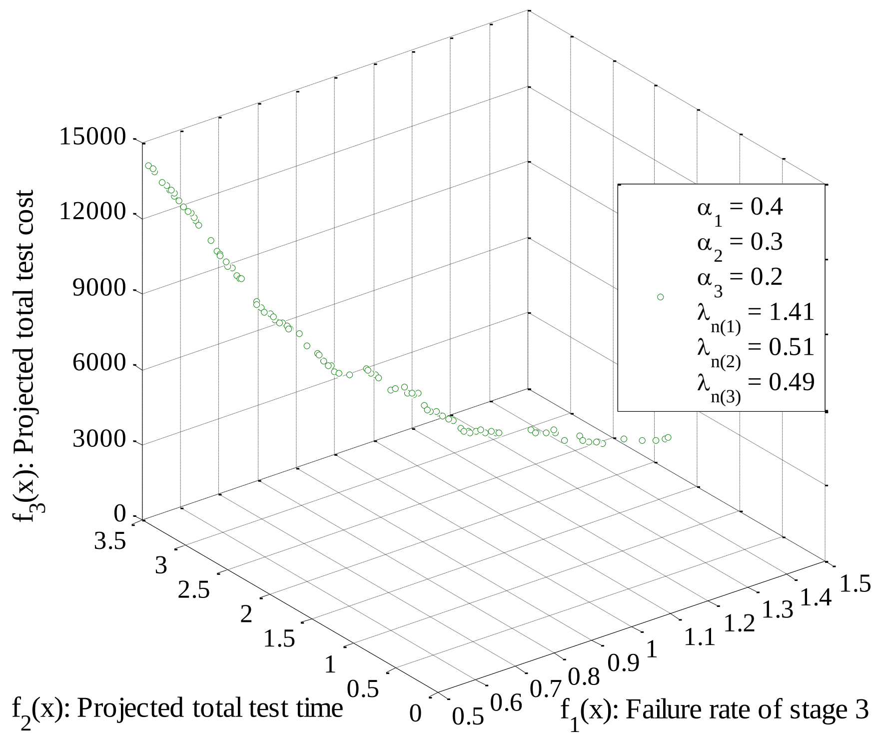

Figure 3.4: Pareto optimal frontier for RGP

Appendix B represents 20 Pareto optimal solutions of the knee area in the Pareto frontier. These solutions are obtained from the original 100 solutions by restricting the failure rate of the final stage to be within [0.9 1.3].

3.5.3. Sensitivity analysis

In order to understand the sensitivity of the objectives with respect to major parameters in reliability growth planning, the growth rates α_i for each stage and the failure rates $\lambda_{n(i)}$ of new contents / technologies in each stage are changed while the other parameters kept constant. Figure

3.5-10 illustrate the sensitivity of objectives to the growth rate changes in each stage ($\alpha_i \pm 0.1$). From the sensitivity analysis results, growth rate change of stage three ($\alpha_3 \pm 0.1$) has bigger effects on the RGP objectives than the growth rate changes of stage two ($\alpha_2 \pm 0.1$) and stage one ($\alpha_1 \pm 0.1$). Similar results are observed for the sensitivity of failure rates with respect to the three RGP objectives. As shown in Figure 3.5-10, the failure rate changes of new contents in stage three ($\lambda_{n(3)} \pm 0.3$) have a bigger impact on RGP objectives values than that by changing failure rates of previous two stages. These sensitivity analysis results indicate that:

(1) Decision makers may be more cautious to introduce new technologies in the pilot development stage since it may delay the overall product development plan;

(2) Earlier efforts may be needed to improve the effectiveness in failure modes discovery and correction such that reliability growth will not be solely dependent on the less controllable final stage growth rate in new product development.

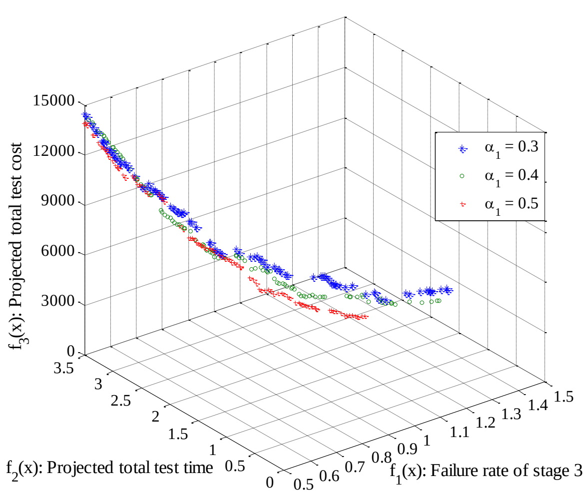

Figure 3.5: Sensitivity analysis of α_1

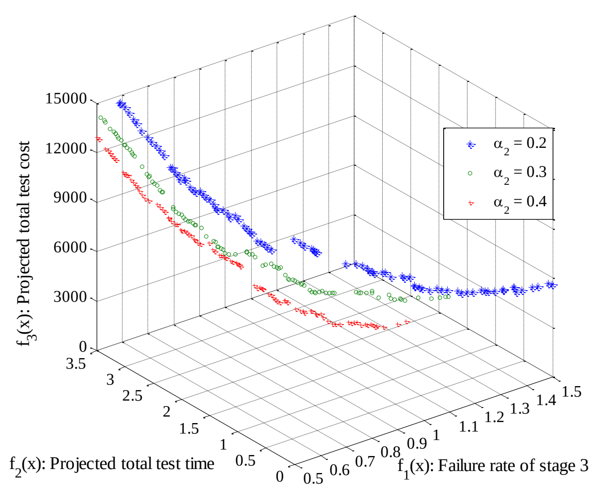

Figure 3.6: Sensitivity analysis of α_2

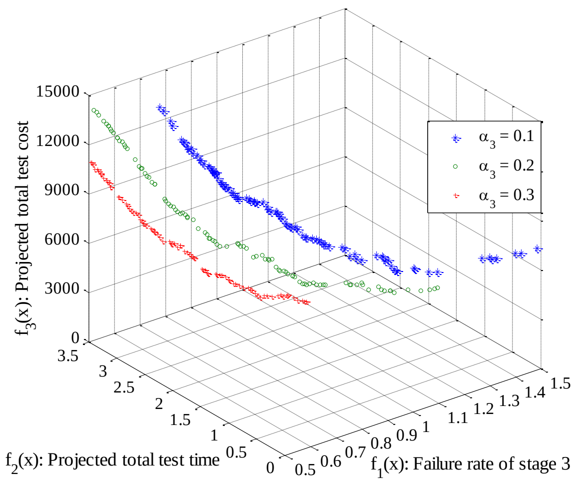

Figure 3.7: Sensitivity analysis of α_3

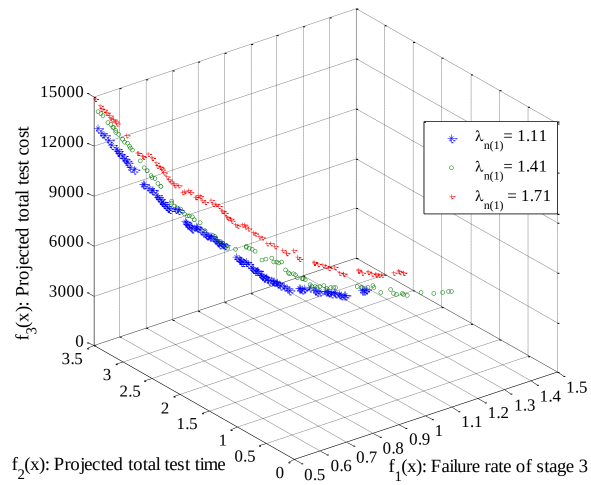

Figure 3.8: Sensitivity analysis of $\lambda_{(n(1))}$

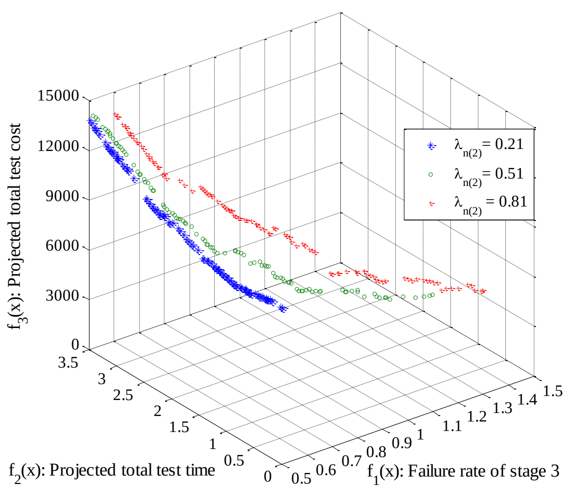

Figure 3.9: Sensitivity analysis of $\lambda_{(n(2))}$

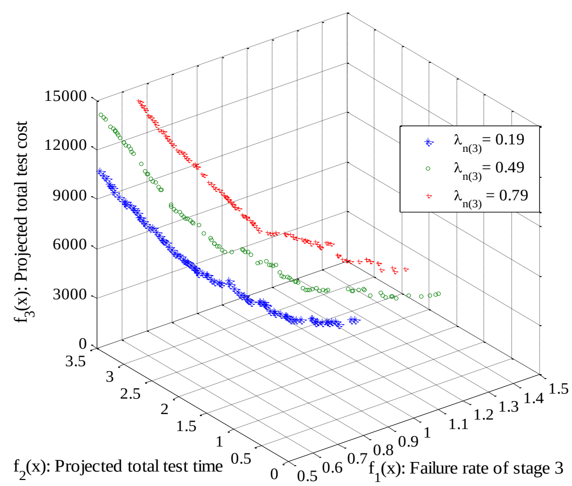

Figure 3.10: Sensitivity analysis of $\lambda_{(n(3))}$

3.5.4. Pareto optimal solutions reduction using DEA

In this section, data envelopment analysis (DEA) is performed to compare the relative efficiency of the 100 Pareto optimal solutions obtained from NSGA-II method such that a few workable Pareto optimal solutions can be presented for decision making for implementing RGP. When applying the DEA method, the Pareto optimal solutions are considered as DMUs. Time and cost objectives are considered as input variables and reliability objective is considered as output variable. To obtain the efficiencies of the 100 DMUs, a linear program needs to be solved for each

DMU. Obviously, as the objective function changes from each linear program to another one, the weights for each DMU may be different. Furthermore, in DEA method, there may be more than one efficient DMU with relative efficiency equal to one, as each individual DMU is trying to select a preferable weight set when evaluating the efficiency of this DMU. The higher relative efficiency value represent that a higher output value can be obtained under a relatively lower amount of weighted inputs.

Both CCR and BCC data envelopment analysis models are applied to evaluate the relative efficiency of the 100 solutions from NSGA-II. Four DMUs were identified as fully efficient ($\theta_0^* = 1 \; and \; S_i^{-*} = S_r^{+*} = 0$) in all four methods including the input-oriented (I-O) and output oriented (O-O) models under both CCR and BCC models. Table 3.3 shows the inputs (cost and time objectives), output (reliability objective) and efficiency results from four different DEA models. The fully efficient, weakly efficient and non-efficient DMUs are represented by (1*), (1), and (0), respectively. The weakly efficient DMUs ($\theta_0^* = 1 \; and \; at \; least \; one \; S_i^{-*} or \; S_r^{+*} \neq 0$) appear just in BCC input and output oriented models. The number of fully efficient DMUs in both BCC input and output oriented models are larger than that from the CCR models, which can be justified by the variable scale to return assumption of BCC model with a more flexible frontier selection [90]. The solutions from CCR input and output oriented models are the same and are plotted in Figure 3.11. The efficient units obtained by BCC input and output oriented models are slightly different and are plotted in Figure 3.12 and Figure 3.13, respectively

In summary, the DEA models can significantly reduce the large Pareto optimal solutions to a few implementable efficient solutions from an economic perspective of considering product development time and cost objectives as inputs and reliability objective, which is measured using mean time between failures (MTBF), as an output. The information from both the original Pareto frontier and the efficient solutions from DEA can be used to select final solutions for RGP implementation. For example, RGP solutions represented by DMUs 15, 86, 96, and 98 (Table 3.3) may be used for the final RGP implantation.

Table 3.3: Inputs, output and results of CCR and BCC models in MO-MS-RGP case study

	Inputs		Output	DEA Models			
DMUs	Time (Yrs.)	Cost ($)	Reliability MTBF (Yrs.)	CCR (I-O)	CCR (O-O)	BCC (I-O)	BCC (O-O)
DMU 01	3.45	15.38	2.13	0	0	1*	1*
DMU 02	0.79	4.56	0.79	0	0	1*	1*
DMU 04	3.28	14.72	2.13	0	0	1*	1*
DMU 07	0.82	4.67	0.83	0	0	1*	1*
DMU 10	2.30	10.87	1.88	0	0	1*	1
DMU 15	**0.88**	**4.91**	**0.89**	**1***	**1***	**1***	**1***
DMU 19	0.80	4.60	0.81	0	0	1*	1*
DMU 26	3.23	14.53	2.10	0	0	0	1*
DMU 31	2.66	12.31	1.96	0	0	0	1
DMU 34	1.76	8.57	1.58	0	0	1*	1*
DMU 36	2.57	11.96	1.94	0	0	0	1
DMU 38	2.25	10.68	1.86	0	0	1*	1*
DMU 48	0.79	4.56	0.79	0	0	1*	1*
DMU 51	2.22	10.56	1.86	0	0	1*	1*
DMU 56	1.83	8.84	1.62	0	0	0	1*
DMU 58	2.29	10.84	1.87	0	0	0	1
DMU 67	1.86	8.95	1.64	0	0	0	1
DMU 69	1.78	8.65	1.60	0	0	1*	1*
DMU 70	2.55	11.88	1.94	0	0	1	1
DMU 86	**1.20**	**6.20**	**1.18**	**1***	**1***	**1***	**1***
DMU 87	1.86	8.94	1.63	0	0	0	1
DMU 96	**1.22**	**6.27**	**1.20**	**1***	**1***	**1***	**1***
DMU 98	**1.21**	**6.23**	**1.19**	**1***	**1***	**1***	**1***

1*: fully efficient, 1: weakly efficient, 0: non-efficient.

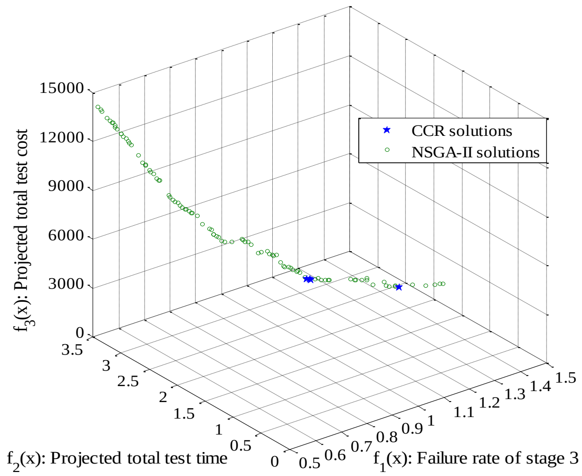

Figure 3.11: DEA solutions, CCR model

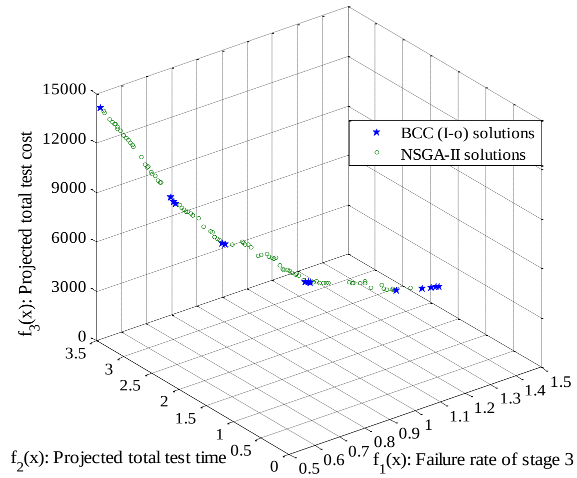

Figure 3.12: DEA results, BCC model

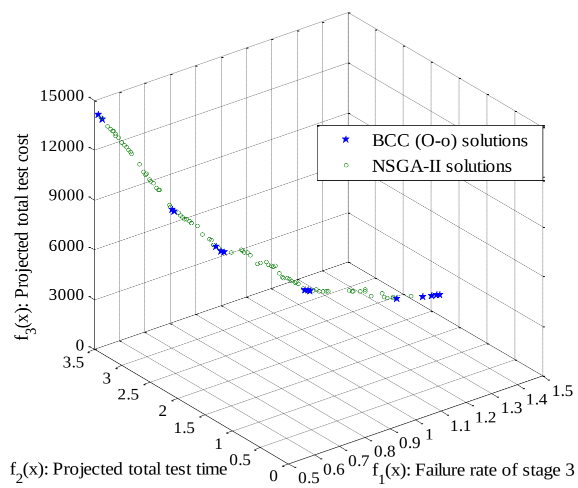

Figure 3.13: DEA solutions, BCC model

3.5.5. Pareto optimal solutions clustering and ranking

After obtaining a set of efficient optimal reliability growth plans, the clustering approaches can be used to provide practical support to the decision makers. The main goal is to assist decision maker to select the choice without precise quantified knowledge about the relative importance of the objective function. In this approach, there is no need to define the preference for the objective functions. Data clustering, which is a multiple criteria partitioning or grouping approach, can be used to group similar objects together in a way that objects with the same cluster have a high degree of similarity. Three objective functions can be considered as features of the efficient optimal solutions as inputs for the clustering tools. Therefore, after obtaining a set of Pareto optimum reliability growth plans in the format of Pareto frontier, the similar reliability growth plans can be categorized into clusters using a clustering method which provides more insights about the obtained Pareto reliability growth plans. By clustering the optimal reliability growth plans, instead of applying the DEA to reduce the optimal solutions, the variety of optimal solutions will be kept since different clusters, each with similar solutions, will be provided, without missing any optimal solutions. In addition, if there are any similar optimal solutions, due to the random process of the evolutionary algorithm in providing optimal solutions, they will be categorized into the same cluster.

After clustering the similar Pareto optimal solutions into homogenous groups, selecting the best reliability growth plan out of the many plan options is still challenging for new product developer. Considering each optimal solution as an alternative with multiple criteria, which can be objective functions, selecting the best Pareto optimal solution can be considered as a multiple criteria decision making (MCDM) problem. MCDM tools can be used in this step to rank the Pareto optimal solution considering all objective functions. Different weights can be assigned to each objective functions considering the decision makers' preferences. As a result, the decision makers can select RGP plans with relative higher rank for reliability growth implementation.

3.6. Summary

A multi-objective and multi-stage reliability growth planning approach is investigated with the goal of providing more accurate and realistic reliability prediction in the early product development stages. Trade-off analysis among multiple product development objectives including development cost, time, and projected reliability can provide decision makers with the insights in terms of the amount and timing of new technologies introduction, optimal testing time and units allocations, and program management efforts for growth rate improvements. The proposed reliability growth planning method does not need new product testing data. The initial failure rates can be estimated by using previous product development experiences as well as failure rates estimation of new technologies based on the Bill of Materials (BOM) in early product design stage. The advantages of the proposed reliability growth planning can provide the management and product development team with critical information such as projected reliability performance, program cost, and product release time in the earlier stage of new product development, no matter how many stages are in the product development process and how many components are in the system. Through sensitivity analysis, the impact of each stage's reliability growth rate uncertainty on the Pareto optimal reliability growth plan is investigated. The growth rates in later stages are more influential to RGP objectives than the growth rates in earlier development stages. Similar results are also observed for the impact of introduced new contents/technologies on projected reliability, cost, and development time objectives. To reduce the large Pareto optimal reliability growth plan alternatives, the DEA method is performed to reduce the number of Pareto solutions by measuring the relative efficiency of each solution, and a workable size of Pareto optimal RGP solutions can further facilitate decision making.

Chapter 4. Design Verification and Validation (V&V) Planning and Optimization for New Product Reliability Improvement

4.1. Introduction

This chapter investigates a method for optimizing product design verification and validation (V&V) planning in the early stages of product development. A V&V plan assigns different engineering activities, e.g., performance testing and engineering analytics, to verify and validate the product design objectives. V&V activities can be derived from the physics-based failure mechanisms through conducting a design failure mode and effect analysis (DFMEA). Each V&V activity is planned to cover specific failure modes for design risk mitigation. The proposed V&V planning optimization model considers the priorities of the failure modes based on their failure occurrence probability and failure consequences. In addition, the sequencing of performing V&V activities and the effectiveness of each V&V activity in reducing failure occurrence rate and improving failure detectability are considered. The objective of the V&V optimization model is to maximize the system reliability improvement by selecting V&V activities and covering the critical failure modes under development time and budget constraints. The sequencing for V&V activities is formulated using an adapted job shop scheduling concept. The set covering problem concept is applied to assure that all critical failure modes are covered by the end of V&V implementation. A numerical example is illustrated to show the application of the proposed model.

4.2. Verification and Validation (V&V) process and activities in product development process

Complex engineering products, such as automobiles and commercial aircraft, require a set of verification and validation activities that satisfy respective legislative requirements governing their use and the increasingly demanding nature of customer aspirations, all within a cost-competitive package and on schedule. In addition, the products themselves are highly complex and designed by large engineering teams spread across many countries and organizations. These factors that, when combined with the exacting requirements, necessitate an effective methodology to plan the implementation of the verification and validation process in the new product development process. As the second phase of this research, a new method is developed to optimize the product design V&V process implementation plan by considering the limited V&V process budget and time, as well as the reliability of the product by the end of implementing the V&V process.

In the different stages of the development lifecycle of a new complex engineering product, e.g. an aircraft or an automobile, there are many verification and validation activities devoted to meet the functional requirements and design objectives of a new product. The planned V&V activities should be conducted in a specific sequence, and under the limited budget. In addition, the product itself is usually highly complex and its failures can be attributed to different failure modes. The product complexity, multiple and possible common and random failure modes, and various V&V options, along with the demanding design requirements and objectives, call for a cost and time effective V&V activities plan which optimally covers all major failure modes of the product. From the reviewed literature, no effective and quantitative methodologies have been well explored for optimal planning of V&V activities.

The challenges in designing an optimal V&V activities plan include:

(1) How to assign a set of V&V activities to cover different failure modes in order to have maximum reliability improvements;

(2) How to optimally allocate the budget to each V&V activity;

(3) How to schedule the V&V activities considering their sequencing requirements;

(4) How to consider the effectiveness of different V&V activities in the failure rate reduction.

Figure 4.1 illustrates the V&V planning process along with the inputs from the DFMEA process and the iterative process for product reliability improvement. As it is summarized, after obtaining reliability risk information of potential failure modes from the DFMEA process including the detectability, severity, and occurrence rate, and the proposed V&V activities to mitigate each failure mode, these V&V activities need to be well planned to meet the requirements such as V&V implementation sequencing, failure mode coverage, development time and budget. The objective for planning the V&V activities is to achieve the maximum product reliability improvement under the NPD requirement and constraints (Figure 4.1).

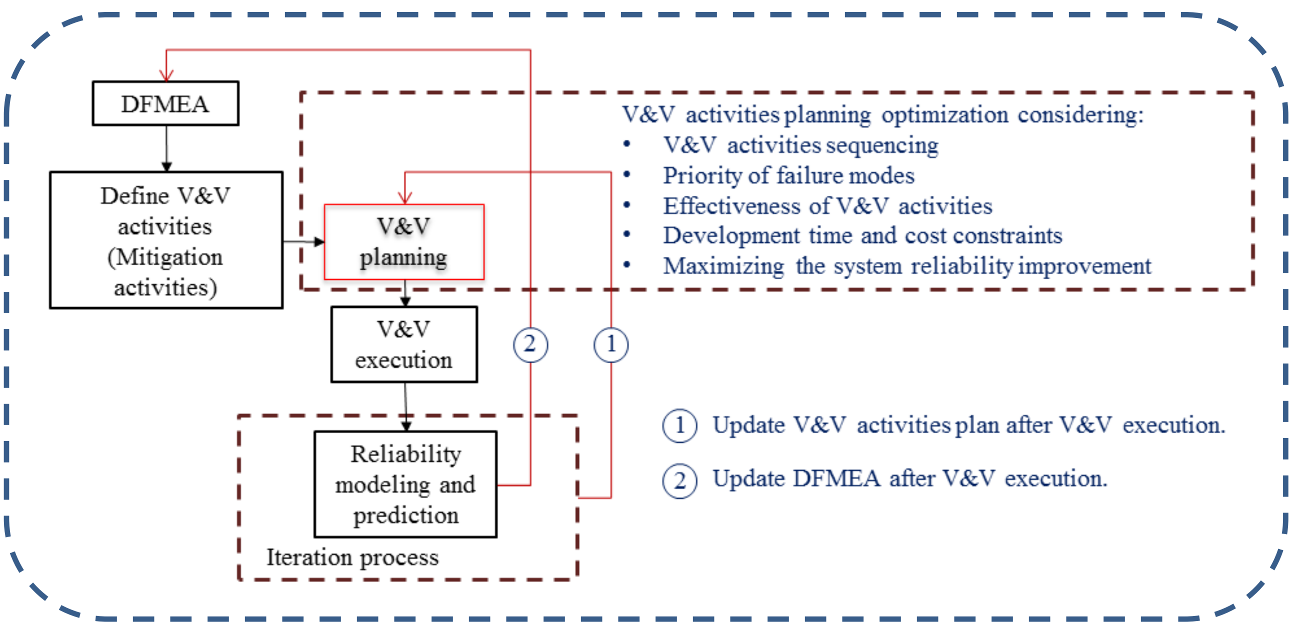

Figure 4.1: A schematic summary of V&V process during NPD for reliability improvement

In this research, a quantitative approach is proposed to model and optimize the planning of the other reliability improvement process in developing a new product, which is the design verification and validation process. The quantitative approach provides an optimal plan for implementing the verification and validation activities when the V&V process time and cost are limited and all failure modes of the new designed product should be mitigated after the implementation of the V&V process. In addition to considering the V&V process time and cost constraints, the sequencing of V&V activities will also be considered in the mathematical modeling. Since implementing each V&V activity will mitigate the failure modes differently, the effectiveness of implementing each V&V activity will be modeled. Furthermore, the priority of failure modes, in terms of the risk priority number, will be considered in the mathematical modeling. The goal is to mitigate the risk priority number of the product's failure modes by implementing an optimal set of V&V activities.

In summary, a list of research tasks to meet the second objective is presented as follows:

- Formulate a mathematical model for the design verification and validation (V&V) process planning to optimize the reliability improvement of the product in terms of mitigating the design failure modes.
- Quantify the effectiveness of V&V activities, the sequencing of V&V activities, and the V&V process limited budget and time in the mathematical modeling.
- Consider the priority of product failure modes in the mathematical modeling.
- Solve the V&V activities planning optimization formulation, which includes a set of V&V activities, to cover all failure modes of the product.

4.3. Modeling of V&V activities planning for reliability improvement

At different stages of the development lifecycle of a new complex engineering product, there are many V&V activities, devoted to meet the functional requirements and design objectives of a new product, which have to be executed on schedule and under the budget constraint by the new product release time. As it was discussed previously, there are some challenges in planning the V&V activities. The first challenge is to prioritize failure modes (f_i), which are assessed during DFMEA process, and will be mitigated by implementing specific V&V activities. For example, failure modes can be prioritized using the Risk Priority Number ($RPN_{i(initial)}$), which is a product of detectability ($D_{i(initial)}$), occurrence ($O_{i(initial)}$), and severity ($S_{i(initial)}$). In this research, detectability, occurrence, and severity are not weighted. When there are multiple alternative V&V activities (v_j) in the V&V process, more effective activities should be selected to be implemented. The effectiveness of V&V activities can be defined by their influence on detectability, i.e., reduction percentage in detecting a potential failure ($\theta_{i,j}$), and their reduction of the occurrence rates of failure modes, i.e., reduction percentage in the failure occurrence rate ($\gamma_{i,j}$). In other words, the most effective set of V&V activities should be selected to provide higher reduction in the RPN of the critical failure modes after completing the V&V process. The updated reliability risk can be expressed as $RPN_{i(new)}$, which is the product of $D_{i(new)}$, $O_{i(new)}$, and $S_{i(new)}$. Scheduling is also a matter of concern in the V&V activities planning. Some V&V activities have to be implemented according to restricted precedence sequences within the total scheduled V&V execution time (T). In addition to sequencing, the limited budget should also be considered and the total cost of the V&V process should not exceed the budget requirement (C). Therefore, there is a trade-off among

V&V cost, duration of the V&V process, and the achievable reliability of the product that is aimed to be improved when planning the V&V activities.

The main assumptions of the proposed V&V planning optimization formulation are summarized as follows. These assumptions are based on the industry practices and the DFMEA handbook [94].

- One failure mode can be mitigated by multiple V&V activities, and one V&V activity can mitigate the reliability risk of multiple failure modes.
- Certain V&V activities have to follow the given implementation sequence. For example, some V&V activities may not be executed during the same time frame due to resources and other constraints. The only case in which V&V activities can have overlap is the case when they start at the same time. Also, there is no time gap between two V&V activities that are in series.
- The execution duration of each V&V activity can be estimated from engineering analysis and all V&V activities should be completed within the product development time T.
- Effectiveness of the j^{th} V&V activity on the failure mode f_i is defined as $\theta_{i,j}$ and $\gamma_{i,j}$, which represent the reduction percentage in detectability $D_{i(initial)}$ and occurrence $O_{i(initial)}$ by implementing a V&V activity v_j, respectively. Unless a design change is made, the severity of a failure mode f_i does not change after performing V&V activities ($S_{i(initial)} = S_{i(new)}$).

Figure 4.2 provides a simple example to illustrate the planning for V&V activities. In this simple example of V&V activities planning, nine potential critical failure modes are identified from the DFMEA process for a newly designed system and six V&V activities are proposed to cover the failure modes. For example, failure mode f_1 will be only mitigated by the 1^{st}, 3^{rd}, and 4^{th} V&V activities; while, the 1^{st} V&V activity will only mitigate failure modes $f_1$and f_3. All V&V activities, except the 4th and 5th V&V activities, which can start at the same time, should be implemented in a specific sequence, e.g., the first activity precedes the second. Due to the limited time and budget and the different levels of effectiveness of these V&V activities, an optimal set of these six V&V activities should be selected for maximum reliability improvement in terms of RPN reduction. In addition, this set of V&V activities should be selected in a way to cover all of these critical failure modes, have the highest risk reduction value in terms of RPN of failure modes, and be implemented in a predefined sequence.

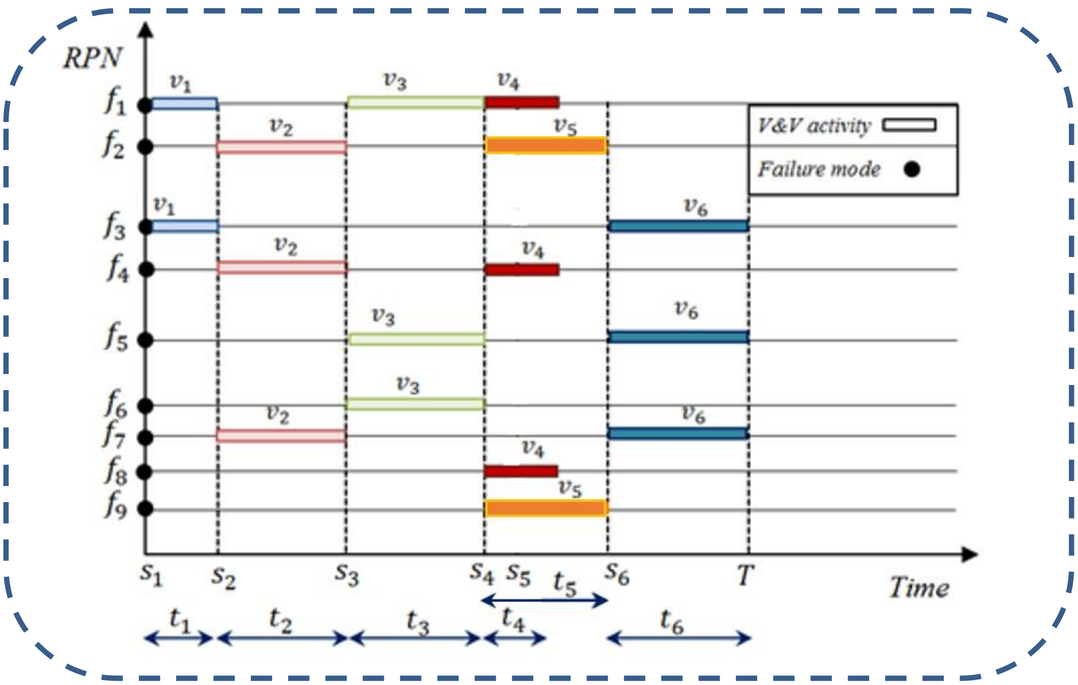

Figure 4.2: An example of V&V activities planning

4.4. Methodology for the V&V planning process

The proposed mathematical model of the V&V planning optimization is described in this section. During the process of V&V activities implementation, it is essential to have all defined critical failure modes mitigated to have a more reliable product. The critical failure modes can be prioritized and then an optimal set of V&V activities should be implemented to cover all failure modes. To formulate the V&V activity planning problem, we first apply the set covering concept to model the requirement that all critical failure modes should be covered by at least one V&V activity. The cost associated with each V&V activity can also be formulated under the set covering formulation (**4.4.1**). Secondly, the job shop scheduling concept is applied to model the constraints of the execution durations and sequencing of V&V activities (**4.4.2**). Lastly, the main goal of V&V activity process which is to reduce the design risk by mitigating critical failure modes, is formulated as the objective function of maximizing the amount of reliability improvements (**4.4.3**). The details are described in the following subsections. Notations that are used in the mathematical modeling of design verification and validation activities planning are provided as follows:

i	Index for failure mode $(i = 1, \dots, n)$
j	Index for V&V activity $(j = 1, \dots, m)$
f_i	Failure mode i $(i = 1, \dots, n)$
v_j	A binary variable indicating if the j^{th} V&V activity is selected ($v_j = 1$) or not ($v_j = 0$)
c_j	Cost of implementing the j^{th} V&V activity
t_j	Duration of the j^{th} V&V activity
s_j	Start time of the j^{th} V&V activity

$\theta_{i,j}$	Reduction percentage in detectability $D_{i(initial)}$ after conducting the j^{th} V&V activity on the failure mode f_i
$\gamma_{i,j}$	Reduction percentage in occurrence $O_{i(initial)}$ after conducting the j^{th} V&V activity on the failure mode f_i
T	Total time for performing the V&V process during NPD
C	Total budget of the V&V process for reliability improvement
$D_{i(initial)}$	Detectability of failure mode f_i before implementing V&V activities
$S_{i(initial)}$	Severity of failure mode f_i before implementing V&V activities
$O_{i(initial)}$	Occurrence of failure mode f_i before implementing V&V activities
$RPN_{i(initial)}$	Risk priority number of failure mode f_i before implementing V&V activities
$D_{i(new)}$	Detectability of failure mode f_i after implementing the selected V&V activities
$S_{i(new)}$	Severity of failure mode f_i after implementing the selected V&V activities
$O_{i(new)}$	Occurrence of failure mode after implementing the selected V&V activities
$RPN_{i(new)}$	Risk priority number of failure mode f_i after implementing selected V&V activities
$a_{i,j}$	A binary variable indicating if j^{th} V&V activity mitigates failure mode i ($a_{i,j} = 1$), or not ($a_{i,j} = 0$).
K_{m*m}	The sequence matric which is an $m * m$ upper triangular matrix with binary variables indicating if j^{th} V&V activity precedes j'^{th} V&V activity ($k_{j,j'} = 1$), or not ($k_{j,j'} = 0$).

4.4.1. Modeling failure mode coverage using the set covering formulation

In this subsection, the model of a general set covering problem (SCP) [95] is briefly described and its application to the proposed V&V activity planning is introduced. Let $M = \{1, \ldots, m\}$ and $N =$

$\{1, \dots, n\}$. Suppose $M_j \subseteq M$ are given, where $j = 1, \dots, m$ are m subsets of M, and c_j is the weight (cost) of the subset j ($j = 1, \dots, m$). A set cover is a subset $W \subseteq \{1, \dots, m\}$, such that $\cup_{j \in W} M_j = M$. In the general model of SCP, the objective is to find a minimum weight (cost) cover (Equation 4.1):

$$\min_{W} \left\{ \sum_{j \in W} c_j : W \ is \ a \ cover \right\} = min \left\{ \sum_{j=1}^{m} c_j v_j : Ax \geq e \, , v_j \in \{0,1\} \right\} \tag{4.1}$$

where A is an $n * m$ incidence matrix with element $a_{ij} = 1$ if $i \in M_j$ and $a_{ij} = 0$ otherwise; e is a vector of 1's ($n * 1$ matrix), v is a vector of v_j ($m * 1$ matrix), and $v_j = 1$, if subset M_j is in the cover, and $v_j = 0$, otherwise. The set covering problem can also be formulated as a problem with the objective function of covering the rows of an n-row, m-column, zero-one matrix (a_{ij}) by a subset of the columns at minimum cost (Equation 4.2).

$$\min \sum_{j=1}^{m} c_j v_j \tag{4.2}$$

The constraint of the set covering model (Equation 4.3) shows that at least one M_j should cover every member of set M, where, $v_j \in \{0,1\}$, and $j = 1, \dots, m$.

$$\sum_{j=1}^{m} a_{i,j} v_j \geq 1, \forall \ i = 1,2, \dots, n \tag{4.3}$$

In the V&V activity planning problem, the goal is to choose a set of V&V activities in such a way that all critical failure modes are covered under the budget constraint. In this case, N is the set of all identified failure modes, and M is the set of all defined V&V activities from DFMEA process, M_j is the set of failure modes that a V&V activity j can mitigate, c_j is the V&V activity execution cost, and W is the set of V&V activities that are selected to be performed (note that it must be a cover of M). Denote v_j as a decision variable indicating if the j^{th} V&V activity is selected or not. As an example shown in Figure 4.2, there are nine failure modes ($n = 9$), and six V&V activities ($m = 6$). Each V&V activity can mitigate certain failure modes as presented in Figure 4.2. In this example, the set of $\{M_3, M_4, M_5, M_6\}$, i.e., $W = \{3,4,5,6\}$, is one set cover in this SCP example. Figure 4.2 can be represented as an incidence matrix $A \in R^{n*m}$, where a_{ij} is equal to 1 if the j^{th} V&V activity mitigates failure mode i, and 0, otherwise.

The objective function in the generic set covering problem is to minimize the cost ($min \sum_{j=1}^{m} c_j v_j$), and it is considered as a constraint ($\sum_{j=1}^{m} c_j v_j \leq C$) in the proposed V&V activities planning model, where c_j represents the cost of implementing the j^{th} V&V activity, $j = 1,2, \dots, m$, and C is the allocated budget for NPD V&V process.

The coverage constraint in Equation 4.3 of this model shows that at least one V&V activity j must cover the failure mode i ($\sum_{j=1}^{m} a_{i,j} v_j \geq 1 \ \forall \ i = 1, \dots, n;\ v_j \in \{0,1\}$). For example, the coverage constraints for the first failure mode ($i = 1$) presented in Figure 4.2 is $v_1 + v_3 + v_4 \ \geq 1$.

4.4.2. Modeling V&V activities sequence using the job shop scheduling

In this subsection, first the general job shop scheduling model is presented, and then, its application in modeling the precedence constraints in V&V activities in the proposed V&V planning model is presented. Note that the notations used in the description of general scheduling model are not related to notations used in the V&V planning description. The goal of a general job shop scheduling problem is to find an optimal schedule for a given collection of jobs (i) where each requires a known sequence of processors (j) that can accommodate one job at a time. Suppose that the processing times are given as t_{ij}, which represent the processing time of job i on the processor

j. The typical decision variables for a job shop scheduling problem are s_{ij} representing the start time of job i on the processor j. The objective function can be to minimize the makespan, i.e., minimize the completion time of the last job. The precedence requirement that job i must complete processing on processor j before starting on processor j' can be expressed as: $s_{ij} + t_{ij} \leq s_{ij'}$. To assure that jobs are not scheduled simultaneously on the same processor, the conflict constraints can be added to the model [96].

In the V&V activity planning, the failure modes are considered as job and V&V activities are considered as processors. Each failure mode (job) can be mitigated by a sequence of V&V activities (processors). Since failure modes can be mitigated simultaneously when a certain V&V activity is implemented, there is no need to define the conflict constraints, which is different than the traditional job shop scheduling. In general, the implementation time of all V&V activities, i.e., makespan of V&V process, should be minimized. The equivalent job shop scheduling objective function for the V&V activity planning can be mathematically modeled as Equation 4.4. Equation 4.4 first finds the maximum completion time of V&V activities. The maximum completion time is also known as the makespan. Then, the makespan of V&V process is minimized for all V&V activities ($j = 1,..,m$).

$$\underset{j=1,..,m}{\text{Min}} \left[Max \left\{\left(s_j + t_j\right)v_j, \forall j = 1, \ldots, m\right\}\right] \tag{4.4}$$

Since the total time of V&V process (T) is limited, and all V&V activities should be implemented under the time constraint, this objective function (Equation 4.4) can be simplified and converted to a constraint presented in Equation 4.5, where T represents the total time for V&V activities implementation. Equation 4.5 guarantees that the completion time for all V&V activities are less than total time of V&V implementation process (T).

$$(s_j + t_j)v_j \leq T, \forall\, j = 1, \dots, m \tag{4.5}$$

The precedence constraints can be presented as Equation 4.6, where $k_{j,j'}$ represents an element of the K_{m*m} upper triangular matrix with binary variables. When $k_{j,j'} = 1$, it indicates that the j^{th} V&V activity precedes j'^{th}V&V activity. If $k_{j,j'} = 0$, it means that the j^{th} V&V activity and the j'^{th}V&V activity can start at the same time.

$$((s_j + t_j)v_j)k_{j,j'} \leq s_{j'},\ \ \forall\, j = 1, \dots, m-1, j' = j+1, \dots, m \tag{4.6}$$

As an example, the proceeding constraints for the first V&V activity in the illustrated problem (Figure 4.2), can be presented as: $((s_1 + t_1)v_1)k_{1,2} \leq\ s_2$ where $k_{1,2} = 1$; $((s_1 + t_1)v_1)k_{1,3} \leq\ s_3$, $k_{1,3} = 1$; and so on. Note that matirx K for the exampel in (Figure 4.2) can be presented as follows. As it is shown, only V&V activities 4 and 5 can start at the same time, and therefore, the value of $k_{4,5} = 0$.

$$K_{6*6} = \begin{bmatrix} - & 1 & 1 & 1 & 1 & 1 \\ - & - & 1 & 1 & 1 & 1 \\ - & - & - & 1 & 1 & 1 \\ - & - & - & - & 0 & 1 \\ - & - & - & - & - & 1 \\ - & - & - & - & - & - \end{bmatrix}$$

The starting time of each V&V activity can be calculated using the following equations (Equation 4.7). Note that the elements of matrix K are used in this equation to deactivate the part of equation when two activities can start at the same time. If j^{th}V&V activity is selected to be implemented, i.e., $v_j = 1$, its starting time can be formulated as:

$$\underset{(j=1,..,m)}{s_j} = s_{j-l} + t_{j-l}k_{j-l\,,j} \text{ if } v_{j-l} = 1, \forall\, l = 1, \dots, m; \text{ where } s_0 = t_0 = 0 \tag{4.7}$$

For example, the starting time of 6th V&V activity in the illustrated problem (Figure 4.2), can be presented as follows. Note that since none of the V&V activities can start at the same time as V&V activity six, according to the sequence matrix K, values of $k_{j,6}$ are all equal to 1 in the following equation.

$$s_6 = \begin{cases} l = 1 \rightarrow s_5 + (t_5 k_{5,6}), \ \ if \ v_5 = 1; \\ l = 2 \rightarrow s_4 + (t_4 k_{4,6}), \ \ if \ v_4 = 1; \\ l = 3 \rightarrow s_3 + (t_3 k_{3,6}), \ \ if \ v_3 = 1; \\ l = 4 \rightarrow \ s_2 + (t_2 k_{2,6}), \ \ if \ v_2 = 1; \\ l = 5 \rightarrow s_1 + (t_1 k_{1,6}), \ \ if \ v_1 = 1; \\ l = 6 \rightarrow s_0 + (t_0 k_{0,6}) = 0 \end{cases}$$

As another example, in order to show how Equation 4.7 works when two activities can start at the same time, the starting time of 5th V&V activities in the illustrated problem (Figure 4.2) is presented as follows. Note that V&V activities four and five can start at the same time, therefore, $k_{4,5}$ is equal to zero, which deactivates the first part of the following equation:

$$s_5 = \begin{cases} l = 1 \rightarrow s_4 + (t_4 k_{4,5}), \ \ if \ v_4 = 1; \\ l = 2 \rightarrow s_3 + (t_3 k_{3,5}), \ \ if \ v_3 = 1; \\ l = 3 \rightarrow: \ s_2 + (t_2 k_{2,5}), \ \ if \ v_2 = 1; \\ l = 4 \rightarrow \ s_1 + (t_1 k_{1,5}), \ \ if \ v_1 = 1; \\ l = 5 \rightarrow s_0 + (t_0 k_{0,5}) = \ 0 \end{cases}$$

For the 4th V&V activity, Equation 4.7 can be presented as follows:

$$s_4 = \begin{cases} l = 1 \rightarrow s_3 + (t_3 k_{3,4}), \ \ if \ v_3 = 1; \\ l = 2 \rightarrow s_2 + (t_2 k_{2,4}), \ \ if \ v_2 = 1; \\ l = 3 \rightarrow: s_1 + (t_1 k_{1,4}), \ \ if \ v_1 = 1; \\ l = 4 \rightarrow s_0 + (t_0 k_{0,4}) = 0 \end{cases}$$

4.4.3. Reliability improvement quantification

One goal of executing V&V activities is to increase the reliability of a system by mitigating the design risk which is usually measured by the Risk Priority Number (RPN) of identified failure modes. Assuming additive effects for all the critical failure modes, the Reliability Improvement Index (RII) for each failure mode i, defined as the ratio of initial risk priority number of a failure mode i ($RPN_{i\,(initial)}$) to the after risk priority number of failure mode i ($RPN_{i\,(new)}$), is adopted to measure the reliability improvement [97]. Note that this RII has the value larger than one and that a higher value implies a larger reliability improvement. Therefore, the total reliability improvement index (RII_{Total}) can be defined as in Equation 4.8:

$$RII_{Total} = \sum\nolimits_{i=1}^{n} RII_i = \sum_{i=1}^{n} \frac{RPN_{i(initial)}}{RPN_{i(new)}} \tag{4.8}$$

where:

$$RPN_{i\,(initial)} = D_{i\,(initial)} O_{i\,(initial)} S_{i\,(initial)} \tag{4.9}$$

$$RPN_{i\,(new)} = D_{i\,(new)} O_{i\,(new)} S_{i\,(new)} \tag{4.10}$$

In real industrial practices, it is common to assume that implementing V&V activity will not affect the severity of failure modes ($S_{i(initial)} = S_{i(new)}$) unless the design changes. Only if a significant change occurs in the design of system, the severity of failure modes can be reduced. The reduction percentage of detectability and occurrence of a failure mode by conducting a V&V activity are denoted as $\theta_{i,j}$ and $\gamma_{i,j}$, respectively. Therefore, the new/updated design risk detectability $D_{i\,(new)}$ and failure occurrence rate $O_{i\,(new)}$ for each $i = 1, \dots, n$ can be presented as Equations (4.11) and (4.12), where v_j is equal to 1, if the j^{th} V&V activity is selected to be implemented, and 0 otherwise.

$$D_{i\,(new)} = D_{i\,(initial)} \prod_{j=1}^{m} (1 - (\theta_{i,j} v_j)) \tag{4.11}$$

$$O_{i\,(new)} = O_{i\,(initial)} \prod_{j=1}^{m} (1 - (\gamma_{i,j} v_j)) \tag{4.12}$$

For example, the reliability improvement index (RII) of the failure mode 1, in the example presented in Figure 4.2, can be expressed as the ratio of $RPN_{1(initial)}$ and $RPN_{1(new)}$, where $RPN_{1(initial)}$ is equal to $D_{1\,(initial)}O_{1\,(initial)}S_{1\,(initial)}$ and $RPN_{1(new)}$ is equal to $D_{1\,(new)}O_{1\,(new)}S_{1\,(new)}$. The relationship between initial and new detectability of failure mode 1 can be presented as: $D_{1\,(new)} = D_{1\,(initial)}\left(1-\left(\theta_{1,1}v_1\right)\right)\left(1-\left(\theta_{1,3}v_3\right)\right)\left(1-\left(\theta_{1,4}v_4\right)\right)$. The same equation can be defined for the occurrence. It is noted that the effect of different V&V activities are assumed to be independently. In reality, after a V&V activity has covered a certain failure mode, the other V&V activity's impact on the same failure mode may be less effective. This can be modeled by defining a decay index for the reduction percentage if multiple activities are applied on the same failure mode.

Based on Figure 4.2, when the first V&V activity is implemented ($v_1 = 1$), it will reduce the detectability and occurrence of failure mode 1 by $\theta_{1,1}$ and $\gamma_{1,1}$, respectively. If the first V&V activity is not selected to be implemented ($v_1 = 0$), it will not affect the detectability and occurrence of failure mode 1. The RII can be defined for each failure mode as it is explained above for failure mode 1.

4.4.4. Mathematical model for the V&V activities planning

The proposed model for V&V activity planning is formulated as follows. The objective function (Equation 4.13) is to maximize the reliability improvement of the product through implementing the planned V&V activities. This objective aims to maximize the summation of Reliability Improvement Index (RII) for all failure modes.

$$MAX: RII_{Total} = \sum_{i=1}^{n} RII_i = \sum_{i=1}^{n} \frac{RPN_{i(initial)}}{RPN_{i(new)}} \tag{4.13}$$

Subject to:

$$\sum_{j=1}^{m} c_j v_j \leq C \tag{4.14}$$

$$\sum_{j=1}^{m} a_{i,j} v_j \geq 1\,, \forall i = 1, \dots, n \tag{4.15}$$

$$(s_j + t_j)v_j \leq T, \forall\, j = 1, \dots, m \tag{4.16}$$

$$((s_j + t_j)v_j)k_{j,j'} \leq s_{j'},\ \ \forall\, j = 1, \dots, m-1, j' = j+1, \dots, m \tag{4.17}$$

where $RPN_{i\,(initial)} = D_{i\,(initial)} O_{i\,(initial)} S_{i\,(initial)}$ and $RPN_{i\,(new)} = D_{i\,(new)} O_{i\,(new)} S_{i\,(new)}$, $D_{i\,(new)} = D_{i\,(initial)} \prod_{j=1}^{m}(1-(\theta_{i,j} v_j))$, and $O_{i\,(new)} = O_{i\,(initial)} \prod_{j=1}^{m}(1-(\gamma_{i,j} v_j))$, and $v_j \in \{0,1\}$ indicating whether the j^{th} V&V activity is selected or not. The first constraint (Equation 4.14) of V&V planning is that the total cost of performing V&V activities should be less than the assigned budget for the V&V activity process. The second constraint (Equation 4.15) confirms that each failure mode is covered by at least one V&V activity through implementing the optimal set of V&V activities. The third constraint (Equation 4.16) guarantees that the completion time of all V&V activities should be less than T, where T represents the total time for V&V activity execution. Since some V&V activities should follow certain precedence, the precedence constraints are also defined for V&V activities. The precedence constraints can be defined for all V&V activities as Equation 4.17.

4.5. Case study: V&V planning and optimization for the next generation engine development

To illustrate the application of the proposed V&V activity planning optimization model, a V&V activity planning example is developed and solved based on a power assembly design when developing a new next generation engine. This example is adopted from an industry engine development application, but the values are transformed for the proprietary consideration. A total number of 15 V&V activities ($m = 15$) is considered which can be categorized into design action, bench test, lab test, and performance test. These V&V activities are proposed during the DFMEA process to mitigate the 25 identified potential failure modes ($n = 25$), which are the most critical failure modes of the system, according to RPN number. The objective of the V&V planning is to select the most effective V&V activities to implement such that maximum reliability improvement can be achieved under limited NPD budget, cost, and other constraints such as V&V implementing sequence and failure mode coverage.

4.5.1. Input data from product design and reliability analysis

This subsection provides the input data for formulating and optimizing the V&V planning for reliability improvement of a new designed power assembly. Part of the information is obtained from the historical data from previous V&V experiences of developing previous generations of similar/other products. The DFMEA report of the new designed product is also extracted for design risk assessment. It should be mentioned that, for the sake of simplicity, all input values are considered as deterministic values in this case study. Considering parameter uncertainty will make the mathematical modeling complex and can be considered as a future research. For each failure mode, the initial detectability $D_{i\,(initial)}$, occurrence rate $O_{i\,(initial)}$, and severity $S_{i\,(initial)}$ can be obtained from the DFMEA process. In this research, the following data about the failure modes are used as shown in Table 4.1. Implementation of each V&V activity can only mitigate the design risk of some failure modes that are described in the incident matrix in Figure 4.3.

There is no precedence restriction for the V&V implementation of v_4 and v_5, meaning that v_4 and v_5 can be started at the same time, if both are selected to implement, and therefore, $j_{4,5} = 0$. Also, v_8 and v_9, and v_{12} and v_{13} can be implemented at the same time. Other V&V activities should be implemented based on the sequences, specified by their subscript indices, i.e., v_{j-1} precedes v_j. The sequence matrix K for the case problem is presented in Figure 4.4. The risk reduction percentage in $D_{i(initial)}$ and $O_{i(initial)}$ after conducting the j^{th} V&V activity on the failure mode f_i, i.e., $\theta_{i,j}$ and $\gamma_{i,j}$, are given in Table 4.2 and Table 4.3, respectively.

The implementation cost and time of each V&V activity are presented in Table 4.4. The total budget (C) for performing the V&V activities is assumed to be $\$470{,}000$ ($C = 470{,}000$) and the total time of implementing V&V process is constrained to be 480 days ($T = 480$). The binary decision variable is v_j which indicates if v_j is selected or not. The optimum set of selected V&V activities should cover all failure modes.

Table 4.1: Initial detectability, occurrence, and severity for each failure mode

i	$D_{i\,(initial)}$	$O_{i\,(initial)}$	$S_{i\,(initial)}$
1	9	1	3
2	5	5	5
3	9	1	9
4	4	4	4
5	6	6	6
6	8	8	8
7	1	2	1
8	9	9	9
9	3	5	5
10	9	2	2
11	2	4	4
12	6	6	6
13	7	6	5
14	1	1	1
15	9	9	9
16	5	5	5
17	5	9	9
18	4	4	4
19	6	6	6
20	1	3	4
21	1	1	1
22	9	6	2
23	3	5	5
24	9	2	1
25	3	3	4

$$
\begin{array}{cc}
 & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} \end{matrix} \\
\begin{matrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \\ f_{10} \\ f_{11} \\ f_{12} \\ f_{13} \\ f_{14} \\ f_{15} \\ f_{16} \\ f_{17} \\ f_{18} \\ f_{19} \\ f_{20} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{24} \\ f_{25} \end{matrix} &
\begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

Figure 4.3: The incidence matrix

$$
\begin{array}{cc}
 & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} \end{matrix} \\
\begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \\ v_{15} \end{matrix} &
\begin{bmatrix}
- & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
- & - & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
- & - & - & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
- & - & - & - & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
- & - & - & - & - & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
- & - & - & - & - & - & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
- & - & - & - & - & - & - & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
- & - & - & - & - & - & - & - & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
- & - & - & - & - & - & - & - & - & 1 & 1 & 1 & 1 & 1 & 1 \\
- & - & - & - & - & - & - & - & - & - & 1 & 1 & 1 & 1 & 1 \\
- & - & - & - & - & - & - & - & - & - & - & 1 & 1 & 1 & 1 \\
- & - & - & - & - & - & - & - & - & - & - & - & 0 & 1 & 1 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & 1 & 1 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & -
\end{bmatrix}
\end{array}
$$

Figure 4.4: The sequence matrix (Matrix K)

Table 4.2: $\theta_{i,j}$: Risk reduction percentage in $D_{i(initial)}$ after conducting the j^{th} V&V activity on the failure mode f_i

j i	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
1	75%	0%	66%	0%	52%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
2	0%	61%	0%	82%	0%	0%	0%	0%	0%	56%	0%	0%	69%	0%	81%
3	0%	0%	0%	0%	0%	63%	0%	62%	0%	0%	0%	0%	0%	0%	0%
4	61%	0%	0%	0%	65%	0%	0%	58%	0%	0%	0%	49%	0%	0%	0%
5	0%	52%	0%	39%	0%	84%	0%	0%	0%	91%	0%	0%	0%	81%	0%
6	0%	0%	52%	0%	0%	0%	0%	0%	56%	0%	0%	0%	0%	0%	0%
7	0%	0%	0%	62%	63%	0%	0%	0%	0%	0%	69%	0%	0%	55%	59%
8	0%	0%	0%	0%	0%	0%	0%	52%	0%	0%	0%	0%	53%	0%	0%
9	0%	0%	36%	0%	0%	86%	0%	0%	0%	56%	0%	0%	0%	0%	0%
10	0%	68%	0%	0%	46%	0%	0%	0%	0%	0%	0%	0%	52%	0%	0%
11	65%	0%	0%	65%	0%	0%	0%	59%	0%	83%	0%	0%	0%	0%	49%
12	0%	0%	62%	0%	0%	0%	36%	0%	0%	0%	29%	0%	0%	0%	0%
13	0%	0%	73%	0%	0%	53%	0%	0%	51%	0%	0%	0%	0%	0%	0%
14	0%	0%	0%	38%	69%	0%	0%	0%	0%	0%	0%	0%	0%	0%	62%
15	26%	0%	92%	0%	0%	0%	0%	71%	0%	0%	0%	0%	0%	0%	0%
16	0%	82%	0%	0%	0%	79%	0%	0%	0%	86%	0%	0%	0%	82%	0%
17	0%	0%	0%	0%	53%	0%	0%	82%	0%	0%	0%	0%	0%	0%	0%
18	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	92%	0%	0%	53%	26%
19	53%	0%	86%	0%	26%	0%	0%	0%	26%	0%	0%	0%	21%	0%	0%
20	0%	0%	0%	91%	0%	0%	0%	0%	0%	0%	82%	0%	0%	0%	0%
21	0%	63%	0%	0%	0%	89%	0%	0%	0%	0%	0%	79%	0%	0%	92%
22	0%	0%	0%	36%	86%	0%	0%	0%	79%	0%	0%	0%	0%	26%	0%
23	22%	0%	79%	0%	0%	0%	0%	0%	0%	0%	53%	0%	0%	0%	0%
24	0%	31%	0%	0%	92%	0%	0%	0%	0%	0%	0%	0%	82%	0%	25%
25	0%	0%	0%	0%	0%	38%	49%	0%	92%	0%	79%	0%	0%	0%	0%

Table 4.3: $\gamma_{i,j}$: Risk reduction percentage in $O_{i(initial)}$ after conducting the j^{th} V&V activity on the failure mode f_i

i \ j	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
1	22%	0%	46%	0%	52%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
2	0%	21%	0%	28%	0%	0%	0%	0%	0%	56%	0%	0%	24%	0%	41%
3	0%	0%	0%	0%	0%	37%	0%	62%	0%	0%	0%	0%	0%	0%	0%
4	61%	0%	0%	0%	21%	0%	0%	58%	0%	0%	0%	49%	0%	0%	0%
5	0%	52%	0%	39%	0%	31%	0%	0%	0%	33%	0%	0%	0%	31%	0%
6	0%	0%	52%	0%	0%	0%	0%	0%	31%	0%	0%	0%	0%	0%	0%
7	0%	0%	0%	62%	37%	0%	0%	0%	0%	0%	24%	0%	0%	55%	19%
8	0%	0%	0%	0%	0%	0%	0%	52%	0%	0%	0%	0%	43%	0%	0%
9	0%	0%	36%	0%	0%	86%	0%	0%	0%	46%	0%	0%	0%	0%	0%
10	0%	68%	0%	0%	46%	0%	0%	0%	0%	0%	0%	0%	52%	0%	0%
11	45%	0%	0%	65%	0%	0%	0%	19%	0%	21%	0%	0%	0%	0%	49%
12	0%	0%	42%	0%	0%	0%	36%	0%	0%	0%	29%	0%	0%	0%	0%
13	0%	0%	26%	0%	0%	53%	0%	0%	51%	0%	0%	0%	0%	0%	0%
14	0%	0%	0%	38%	24%	0%	0%	0%	0%	0%	0%	0%	0%	0%	62%
15	26%	0%	42%	0%	0%	0%	0%	71%	0%	0%	0%	0%	0%	0%	0%
16	0%	28%	0%	0%	0%	79%	0%	0%	0%	46%	0%	0%	0%	21%	0%
17	0%	0%	0%	0%	53%	0%	0%	28%	0%	0%	0%	0%	0%	0%	0%
18	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	21%	0%	0%	43%	26%
19	53%	0%	46%	0%	26%	0%	0%	0%	26%	0%	0%	0%	21%	0%	0%
20	0%	0%	0%	33%	0%	0%	0%	0%	0%	0%	31%	0%	0%	0%	0%
21	0%	37%	0%	0%	0%	89%	0%	0%	0%	0%	0%	79%	0%	0%	92%
22	0%	0%	0%	36%	86%	0%	0%	0%	79%	0%	0%	0%	0%	26%	0%
23	22%	0%	79%	0%	0%	0%	0%	0%	0%	0%	21%	0%	0%	0%	0%
24	0%	31%	0%	0%	42%	0%	0%	0%	0%	0%	0%	0%	28%	0%	25%
25	0%	0%	0%	0%	0%	38%	49%	0%	92%	0%	79%	0%	0%	0%	0%

Table 4.4: Cost and duration of each V&V activity

j	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
c_j **($1000)**	100	80	82	65	96	81	82	85	41	23	95	93	54	63	75
t_j **(day)**	25	50	71	56	56	52	48	99	99	82	74	53	53	58	50

4.5.2. Mathematical modeling

The objective function is to maximize the reliability improvement which is represented as the summation of the ratios of initial and new RPN for all failure modes, i.e., the reliability improvement index (RII). In this numerical example, the objective function is formulated as $RII_{Total} = \sum_{i=1}^{25} RII_i = \sum_{i=1}^{25} \frac{RPN_{i(initial)}}{RPN_{i(new)}}$, in which $RPN_{i\,(initial)}$ and $RPN_{i\,(new)}$ are expressed in Equations (4.9) and (4.10), respectively. For simplicity, only the relationships between design risk detectability of $D_{i\,(new)}$ and $D_{i\,(initial)}$ for only the first and last five failure modes are presented in Table 4.5. The similar equations can be obtained for the relationship between failure occurrence of $O_{i\,(new)}$ and $O_{i\,(initial)}$, by replacing θ_{ij} with γ_{ij}.

Table 4.5: $\mathrm{D}_{\mathrm{i\,(new)}}$ for each failure mode

$\boldsymbol{i}$	$\boldsymbol{D_{i\,(new)}}$
$i = 1$	$D_{1\,(new)} = D_{1\,(initial)} * \left(1-(\theta_{11} * v_1)\right) * \left(1-(\theta_{13} * v_3)\right) * (1-(\theta_{15} * v_5))$
$i = 2$	$D_{2\,(new)} = D_{2\,(initial)} * \left(1-(\theta_{22} * v_2)\right) * \left(1-(\theta_{24} * v_4)\right) * \left(1-(\theta_{2\,10} * v_{10})\right) * \left(1-(\theta_{2\,13} * v_{13})\right) * \left(1-(\theta_{2\,15} * v_{15})\right)$
$i = 3$	$D_{3\,(new)} = D_{3\,(initial)} * \left(1-(\theta_{36} * v_6)\right) * \left(1-(\theta_{38} * v_8)\right)$
$i = 4$	$D_{4\,(new)} = D_{4\,(initial)} * \left(1-(\theta_{41} * v_1)\right) * \left(1-(\theta_{45} * v_5)\right) * \left(1-(\theta_{48} * v_8)\right) * \left(1-(\theta_{4\,12} * v_{12})\right)$
$i = 5$	$D_{5\,(new)} = D_{5\,(initial)} * \left(1-(\theta_{52} * v_2)\right) * \left(1-(\theta_{54} * v_4)\right) * \left(1-(\theta_{56} * v_6)\right) * \left(1-(\theta_{5\,10} * v_{10})\right) * \left(1-(\theta_{5\,14} * v_{14})\right)$
...	....
$i = 21$	$D_{21\,(new)} = D_{21\,(initial)} * \left(1-(\theta_{21\,2} * v_2)\right) * \left(1-(\theta_{21\,6} * v_6)\right) * \left(1-(\theta_{21\,12} * v_{12})\right) * \left(1-(\theta_{21\,15} * v_{15})\right)$
$i = 22$	$D_{22\,(new)} = D_{22\,(initial)} * \left(1-(\theta_{22\,4} * v_4)\right) * \left(1-(\theta_{22\,5} * v_5)\right) * \left(1-(\theta_{22\,10} * v_{10})\right) * \left(1-(\theta_{22\,14} * v_{14})\right)$
$i = 23$	$D_{23\,(new)} = D_{23\,(initial)} * \left(1-(\theta_{23\,1} * v_1)\right) * \left(1-(\theta_{23\,3} * v_3)\right) * (1-(\theta_{23\,11} * v_{11}))$
$i = 24$	$D_{24\,(new)} = D_{24\,(initial)} * \left(1-(\theta_{24\,2} * v_2)\right) * \left(1-(\theta_{24\,5} * v_5)\right) * \left(1-(\theta_{24\,13} * v_{13})\right) * \left(1-(\theta_{24\,15} * v_{15})\right)$
$i = 25$	$D_{25\,(new)} = D_{25\,(initial)} * \left(1-(\theta_{25\,6} * v_6)\right) * \left(1-(\theta_{25\,7} * v_7)\right) * \left(1-(\theta_{25\,9} * v_9)\right) * \left(1-(\theta_{25\,11} * v_{11})\right)$

The first set of constraints for this V&V planning optimization problem is the cost constraint, which can be defined based on Equation (4.14) as the following: $c_1 v_1 + c_2 v_2 + \cdots + c_{15} v_{15} \leq 470{,}000$. The coverage constraints (Equation 4.15) should also be formulated for each failure mode. Table 4.6 lists the coverage constraints for the first and last five failure modes.

Table 4.6: Coverage constraint for each failure mode

i	**The coverage constraint**
$i = 1$	$v_1 + v_3 + v_5 \geq 1$
$i = 2$	$v_2 + v_4 + v_{10} + v_{13} + v_{15} \geq 1$
$i = 3$	$v_6 + v_8 \geq 1$
$i = 4$	$v_1 + v_5 + v_8 + v_{12} \geq 1$
$i = 5$	$v_2 + v_4 + v_6 + v_{10} + v_{14} \geq 1$
...	...
$i = 21$	$v_2 + v_6 + v_{12} + v_{15} \geq 1$
$i = 22$	$v_4 + v_5 + v_9 + v_{14} \geq 1$
$i = 23$	$v_1 + v_3 + v_{11} \geq 1$
$i = 24$	$v_2 + v_5 + v_{13} + v_{15} \geq 1$
$i = 25$	$v_6 + v_7 + v_9 + v_{11} \geq 1$

According to Equation (4.16), the time constraint of the developed numerical example can be formulated for all V&V activities as follows: $(s_1 + t_1) * v_1 \leq 480$; $(s_2 + t_2) * v_2 \leq 480$, and so on, when $T = 480$. Also, the precedence constraints formulated based on Equation (4.17) can be presented for all V&V activities.

4.5.3. Numerical results

The constrained, nonlinear single objective formulation with binary decision variables (v_j) is solved using a generalized reduced gradient (GRG) method, which has been proven to be one of the most efficient and reliable nonlinear optimization methods [98]. The algorithm also uses a multi-start point concept [99], which leads to finding the globally optimal solutions. The problem is solved on an Intel Core i7 personal computer with 8G RAM. The computation time to get the final solutions for this specific case study problem, with 15 decision variables, is around 32 seconds.

The values for v_j and S_j for each V&V activity are presented in Table 4.7. The results show that six V&V activities, including v_3, v_8, v_{11}, v_{13}, v_{14}, and v_{15}, out of the proposed 15 V&V activities are selected for implementation to optimally mitigate all critical failure modes. The objective function value is obtained as: $RII_{Total} = \sum_{i=1}^{25} RII_i = 850.132$. In addition to RII_{Total}, the reduction in total RPN is calculated as: $Sum\ (RPN\ initial) - Sum\ (RPN\ new) = 3635.907$. Furthermore, the summation of ratio of the initial and new detectability, as well as occurrence, for all failure modes is calculated as: $\sum_{i=1}^{25} \frac{O_{initial}}{O_{new}} + \sum_{i=1}^{25} \frac{D_{initial}}{D_{new}} = 74.26 + 213.57 = 287.83$. It should be mentioned that the problem is solved by considering the three possible objective functions, individually, and in all cases, the same optimal solutions are provided in this case study.

Total cost of implementing the selected six V&V activities is $454,000 and the total implementation time is obtained as 405 days.

Table 4.7: Decision variables (v_j) and the starting time of each V&V activity

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
v_j	0	0	1	0	0	0	0	1	0	0	1	0	1	1	1
S_j	-	-	0	-	-	-	-	71	-	-	170	-	244	297	355

The summary of the results for each failure mode, including the new detectability and occurrence rates, and the initial and new risk priority numbers, is presented in Table 4.8. As it is shown in Table 4.8, the average of reliability improvement percentage, in terms of the reduction percentage in the risk priority number for all failure modes, is approximately 86.8%. A schematic view of the optimal V&V activities plan is presented in Figure 4.5.

Table 4.8: Summary of the V&V activities planning results

$\boldsymbol{i}$	$\boldsymbol{RPN_{i(initial)}}$	$\boldsymbol{D_{i\,(new)}}$	$\boldsymbol{O_{i\,(new)}}$	$\boldsymbol{RPN_{i(new)}}$	**RPN reduction (%)**
1	27	3.0600	0.5400	4.957	81.6%
2	125	0.2945	2.2420	3.301	97.4%
3	81	3.4200	0.3800	11.696	85.6%
4	64	1.6800	1.6800	11.290	82.4%
5	216	1.1400	4.1400	28.318	86.9%
6	512	3.8400	3.8400	117.965	77.0%
7	2	0.0572	0.5540	0.032	98.4%
8	729	2.0304	2.4624	44.997	93.8%
9	75	1.9200	3.200	30.720	59.0%
10	36	4.3200	0.9600	8.294	77.0%
11	32	0.4182	1.6524	2.764	91.4%
12	216	1.6188	2.4708	23.998	88.9%
13	210	1.8900	4.4400	41.958	80.0%
14	1	0.3800	0.3800	0.144	85.6%
15	729	0.2088	1.5138	2.845	99.6%
16	125	0.9000	3.9500	17.775	85.8%
17	405	0.9000	6.4800	52.488	87.0%
18	64	0.1113	1.3328	0.593	99.1%
19	216	0.6636	2.5596	10.191	95.3%
20	12	0.1800	2.0700	1.490	87.6%
21	1	0.0800	0.0800	0.006	99.4%
22	108	6.6600	4.4400	59.141	45.2%
23	75	0.2961	0.8295	1.228	98.4%
24	18	1.2150	1.0800	1.312	92.7%
25	36	0.6300	0.6300	1.588	95.6%
	$Sum = 4115$			$Sum = 479$	$Avg. =$ 86.8%

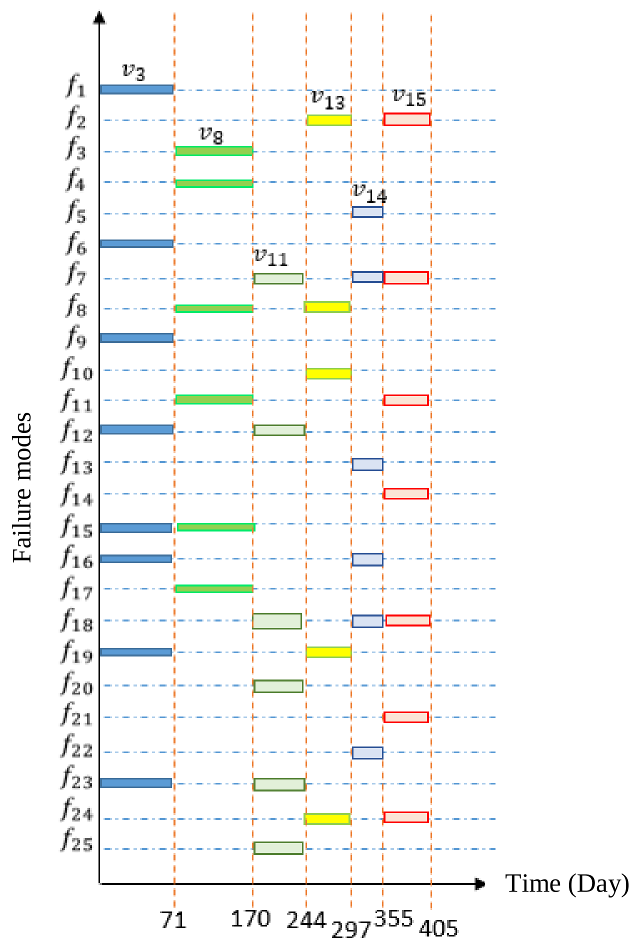

Figure 4.5: A Schematic view of the optimal V&V activities plan

4.6. Comparative analysis and performance evaluation

The quantitative approach to plan the V&V activities in the new product development process, proposed and applied in this research, is the first quantitative approach in the V&V planning literature in which all challenges of V&V activities planning, including reliability improvement, failure coverage, effectiveness of V&V activities, scheduling, and budgeting, are considered. Although other quantitative approaches used in the V&V planning literature, such as PERT, capture some features of V&V activities planning such as scheduling and budgeting, but these approaches lack in modeling the reliability improvement, which is the main goal of V&V activities planning. In this section, three approaches in V&V activities planning are used to solve the case problem and the results are compared with the proposed V&V planning model provided in this research. These three approaches include: 1) PERT approach in which only sequencing and scheduling of V&V activities are considered; 2) Cost-oriented V&V planning approach in which the total cost of V&V activities implementation is minimized as objective function and 3) Time-oriented V&V planning approach in which total time of V&V activities implementation is minimized. Note that there is no reliability improvement maximization in the mentioned three approaches except a constraint that defined to have the failure coverage.

4.6.1. Planning V&V activities using PERT

In order to compare the results obtained from the model proposed in this research with the traditional model in the literature, PERT approach is used to model the V&V activities planning problem in the case study (described in Section 4.5), knowing that some inputs data, including reliability-related data obtained from DFMEA cannot be used in the PERT approach. Considering the sequencing and duration of each V&V activity, the V&V plan provided by PERT is presented in Figure 4.6. Note that considering the sequencing of V&V activities, as well as the limited time and budget, PERT approach is only able to provide a plan for V&V activities that satisfies the sequencing constraints, while it is an infeasible solution in terms of satisfying time and cost constraints. Table 4.9 Provides the duration, and starting and finishing times of V&V activities. Total cost of implementing V&V activities in equal to $1,115,000, while total time is obtained as 718 days.

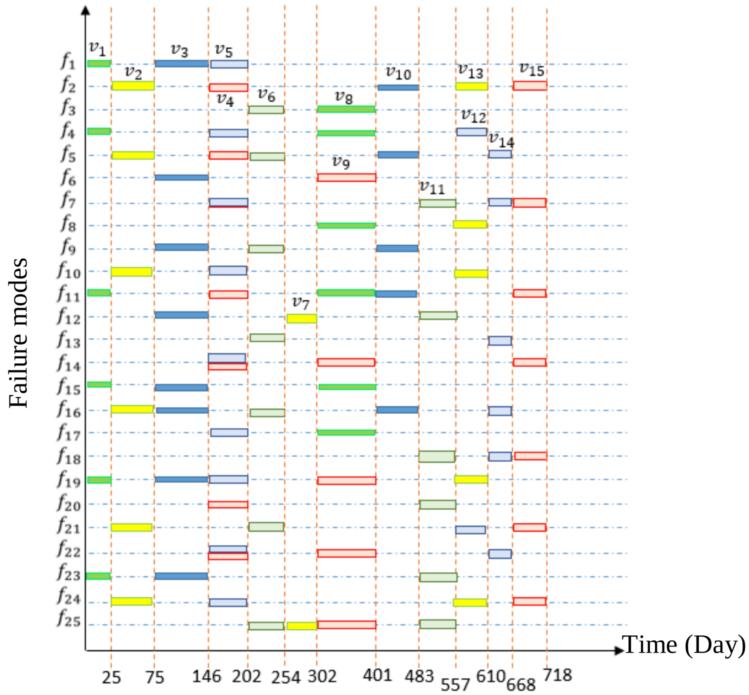

Figure 4.6: A schematic view of V&V plan obtained from PERT approach

Table 4.9: The V&V process schedule obtained from PERT approach

V&V activity	Duration	Start	Finish
1	25	0	25
2	50	25	75
3	71	75	146
4	56	146	202
5	56	146	202
6	52	202	254
7	48	254	302
8	99	302	401
9	99	302	401
10	82	401	483
11	74	483	557
12	53	557	610
13	53	557	610
14	58	610	668
15	50	668	718

4.6.2. Cost-oriented V&V planning approach

The proposed V&V planning model, presented in Equation 4.13-17, is used to model and solve the case problem when the original proposed model has been modified. First, the objective function of the V&V panning model is considered as minimizing the total cost of V&V implementation while the reliability improvement is not considered in the model. The modified model, called cost-oriented approach, is presented in Equation 4.18.

$$\text{Min} \sum_{j=1}^{m} c_j v_j$$

Subject to:

$$\sum_{j=1}^{m} a_{i,j} v_j \geq 1, \forall i = 1, \dots, n \tag{4.18}$$

$$(s_j + t_j) v_j \leq T, \forall\, j = 1, \dots, m$$

$$((s_j + t_j) v_j) k_{j,j'} \leq s_{j'}, \ \forall\, j = 1, \dots, m-1, j' = j+1, \dots, m$$

The optimal V&V plan obtained by solving this optimization problem is presented in Figure 4.7 and Table 4.10. Total cost is minimized at \$413,000 with the duration of V&V implementation as 419 days. The average of reliability improvement percentage, in terms of the reduction percentage in the risk priority number for all failure modes, is 49.8 %.

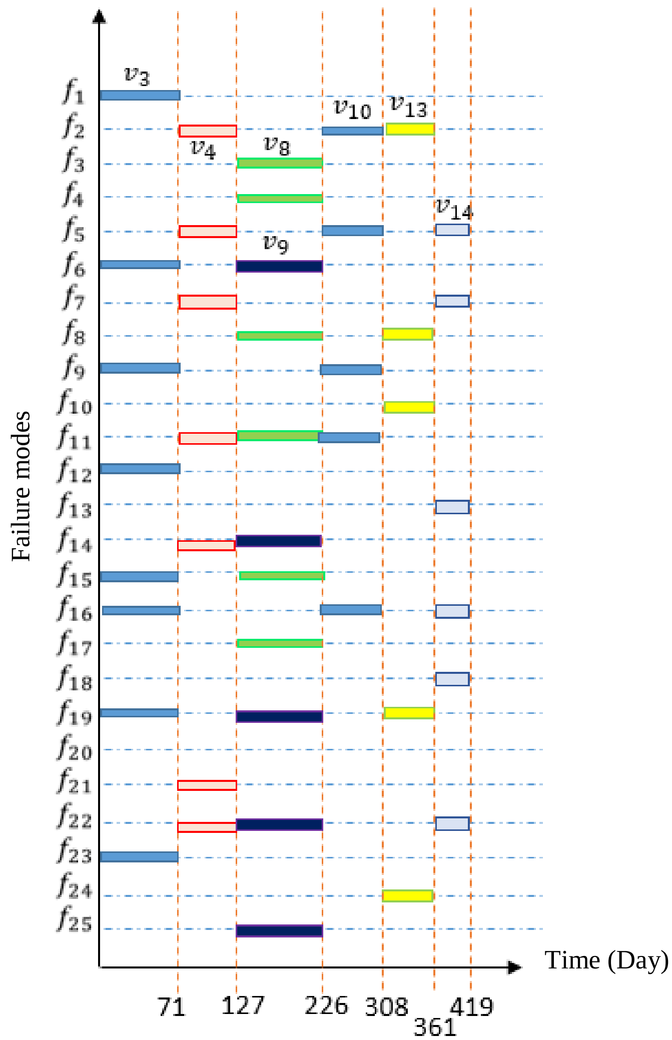

Figure 4.7: A schematic view of V&V plan obtained from cost-oriented V&V planning approach

Table 4.10: The V&V process schedule obtained from cost-oriented V&V planning approach

V&V activity	Duration	Start	Finish
3	71	0	71
4	56	71	127
8	99	127	226
9	99	127	226
10	82	226	308
13	53	308	361
14	58	361	419

4.6.3. Time-oriented V&V planning approach

In addition to considering cost-oriented V&V planning approach, the case problem is solved when the V&V planning is considered to be time-oriented. Therefore, the V&V planning model is modified and solved in a way that total time is minimized when the reliability improvement is removed from the model, as presented in Equation 4.19. The case study is modeled and solved using the modified V&V planning model. Results are presented in Figure 4.8 and Table 4.11. Using the time-oriented V&V planning approach, total cost of V&V implementation is obtained as $568,000, while total time of V&V implementation process is minimized to 357 days. Note that the solution obtained in this approach is an infeasible solution since its cost value is higher than the assigned budget (C) for implementing the V&V process. The average reliability improvement percentage, in terms of the reduction percentage in the risk priority number for all failure modes, is approximately 57.9 %.

$$Min(\underset{j=1,..,m;\ i=1,...,n}{Max}\{(s_j + t_j)\ v_j\})$$

Subject to:

$$\sum_{j=1}^{m} a_{i,j} v_j \geq 1\,, \forall i = 1, \dots, n \tag{4.19}$$

$$\sum\nolimits_{j=1}^{m} c_j v_j \leq C$$

$$((s_j + t_j)v_j)k_{j,j'} \leq s_{j'},\ \ \forall\, j = 1, \dots, m-1,\ j' = j+1, \dots, m$$

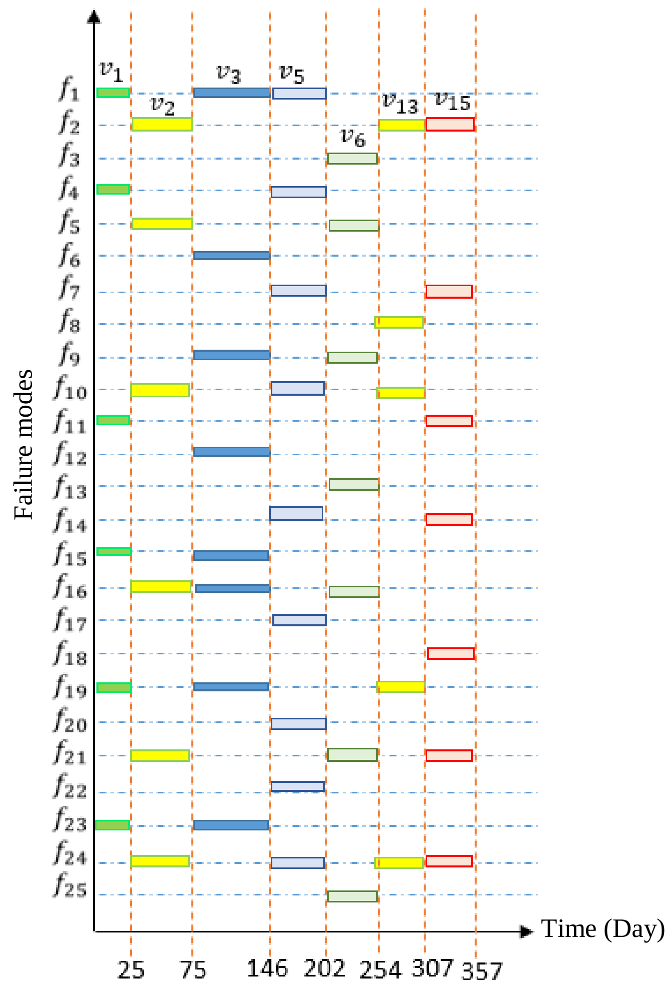

Figure 4.8: A schematic view of V&V plan obtained from time-oriented V&V planning approach

Table 4.11: The V&V process schedule obtained from time-oriented V&V planning approach

V&V activity	Duration	Start	Finish
1	25	0	25
2	50	25	75
3	71	75	146
5	56	146	202
6	52	202	254
13	53	254	307
15	50	307	357

4.6.4. Summary of comparative analysis

The results of all approaches in planning V&V activities are provided in Table 4.12. Comparing the results obtained from four approaches, the maximum reliability improvement is obtained when the PERT approach is used to model and solve the case problem. This result makes sense since all V&V activities are implemented in this case, however, total time and cost of V&V implementation are beyond the assigned time and budget of V&V implementation. Therefore, the solution provided by PERT is an infeasible solution. Comparing two feasible solutions, total cost of V&V implementation in the reliability-oriented approach is higher than total cost when the cost-oriented approach is used, however, the reliability improvement obtained from the cost-oriented approach is significantly lower than the reliability-oriented approach, since the effectiveness of V&V activities is not considered in selecting the optimal set of V&V activities. Total time of V&V implementation has its minimum value when the total time is minimized in the time-oriented approach, however, total cost of V&V activities implementation is significantly higher than other two approaches. In addition, the cost of V&V activities implementation in the time-oriented approach is higher than assigned budget for V&V activities implementation, and therefore, the solution generated by this approach is an infeasible solution. As the main objective of V&V activities implementation is to maximize the reliability of the product, the reliability-oriented approach can be suggested as the most effective approach to achieve this critical goal in new product development process. However, depending on the defined objectives of new product development process, other two approaches including time and cost-oriented approaches, can be utilized in the V&V activities planning process.

Table 4.12: Summary of comparison of three approaches in planning V&V activities

Approach	**Summary of results**				**Feasibility of solution**
	Selected V&V activities	Total time (days)	Total cost ($)	Reliability improvement (RPN reduction %)	
PERT	All	718	1,115K	100%	Infeasible
Cost-oriented	3, 4, 8, 9, 10, 13, 14	419	413K	49.8%	Feasible
Time-oriented	1, 2, 3, 5, 6, 13, 15	357	568K	57.9%	Infeasible
Reliability-oriented (Proposed approach)	3, 8, 11, 13, 14, 15	405	454K	86.8%	Feasible

4.7. Summary

To gain competitive advantages, firms that design and develop new complex products seek to increase the efficiency of their new product development (NPD) processes. The verification and validation (V&V) process is one of the main processes in the early stages of the NPD, which includes a series of engineering activities defined to meet design objectives and performance requirements, such as a desired reliability level. This research introduces a novel mathematical model to optimize the V&V activities planning for improving the reliability of a new designed product while taking into account constraints in development time and budget. There are different potential failure modes of a new developed product, which can be identified through a design failure mode and effect analysis (DFMEA). To mitigate these potential failure modes, various V&V activities are proposed to be implemented during early stages of a new product development process. These activities can be simple engineering calculations, complex simulation studies, and costly subsystem tests. Thus, the effectiveness of these V&V activities can be very different. The proposed V&V planning approach optimally selects a set of V&V activities to mitigate all failure modes for maximum reliability improvement. The set covering model is adopted to assure that each failure mode is covered with at least one V&V activity under limited budget constraint. The time constraint and sequencing of V&V activities are modeled using the job shop scheduling concepts. A numerical example of V&V activities planning for a new designed power assembly is developed, modeled, and solved to show the application of the proposed V&V planning model. The proposed mathematical model for V&V activities planning can benefit the management and product development team with critical information, such as the amount of improvement in reliability of the product by implementing a specific set of V&V activities, the budget, and time schedule for covering all critical failure modes through the V&V process.

Chapter 5. Conclusion and Future Works

5.1. Conclusion

No business can continue to offer the same unchanged product. To stay ahead of their competitors, companies need to develop new products which bring in higher sales, increased customer loyalty, and ultimately higher profits. New product developers have always faced some challenges to keep the balance between the key elements of the new product development (NPD) process which are the quality/reliability of the developed product, the new product development process time, and the cost of developed products. Recent practices in developing a new complex product have been mostly plagued with cost overrun, schedule delays, and quality issues during the design stages. In order to deal with these issues, this research is focused on the modeling and optimization of planning the two reliability improvement processes, reliability growth process and design verification and validation process, which both are usually implemented during early design stages of developing a product.

Reliability growth of a product has always been the major consideration in the early stages of developing a new product since product design modifications are easy to implement and need shorter time and cost. Most of the existing research has only focused on improving the reliability of the product at the later stages of development, when the product is on the field and the test data are available. In addition, the existing research in the literature modeled the reliability growth of a developed product as a single stage; however, the reliability growth process usually involves multiple stages during multiple developing stages of a new product. Moreover, most of the research in the literature only considered one of the objectives of developing a new product development process, which include reliability of the product, and NPD cost and time. The first part of this research investigated a mathematical model for reliability growth planning while developing a new product. The reliability growth model included three objective functions, including maximizing the reliability of the product, minimizing the development time, and minimizing the development cost. Since the reliability growth programs are mostly implemented during several stages, the proposed model formulated the reliability growth in a way that multiple stages of the reliability growth process are integrated. The proposed multi-objective, multi-stage reliability growth model was optimized using an integrated multi-objective optimization. The optimization process started with obtaining optimal solutions using the fast non-dominated sorting genetic algorithm (NSGA-II) and then comparing the optimal solutions in terms of relative efficiency using the data envelopment analysis (DEA). As a case study, the reliability growth plan of the next generation engine was modeled and optimized and a trade-off analysis between product reliability, development cost, and time were presented.

In addition to the reliability growth planning, product design verification and validation (V&V) activities planning process plays a critical role in improving the reliability of a new product. Identified failure modes of a new product, recognized by conducting the design failure modes and effect analysis (DFMEA), should be mitigated by implementing a set of V&V activities. V&V activities have different levels of effectiveness in terms of mitigating the failures' effects, should be implemented on schedule, in a specific sequence, and under a limited budget. The existing V&V planning models in the literature have ignored at least one or two perspectives of V&V process, such as: the V&V activities time and cost constraints, effectiveness and sequencing of V&V activities, critical failure coverage, etc. In the second part of this research, a novel quantitative approach to model and optimize the verification and validation activities planning was proposed.

The model provided the optimal set of V&V activities to be implemented in which all critical failure modes are covered on schedule and under budget. Effectiveness levels of V&V activities were considered in the studied model in a way that the most effective V&V activities, in terms of their reduction percentage on failures' detectability and occurrence, were selected to be implemented. As a case study, the investigated V&V activities planning model was applied in a numerical example of a power assembly design when developing a new next generation engine.

5.2.Future work

The first part of this research provided a new approach in modeling and optimizing the reliability growth planning process in developing a new complex engineering system, while multiple objectives of new product development process are considered and multiple stages of developing a new product are integrated. There are some potential extensions to the first part of this research which are listed as follows:

- Prior knowledge from product development processes, such as the expert's judgment and historical data, can be investigated and integrated in RGP modeling under a Bayesian inference framework.
- Different sources of uncertainties in the RGP process, such as uncertainties involved in failure rates, reliability growth rate, reliability growth time and cost, etc., can be included in the proposed mathematical RGP model using the fuzzy and grey theories.
- Other quantitative methods, such as dynamic programming, can be applied to integrate multiple stages of developing a new product in modeling and optimizing RGP process.

- In addition to the test unit and test time for each sub-system, other decision variables, such as the percentage of technology introduction, can be investigated in the proposed RGP model.
- Other reliability improvement processes, such as accelerated life tests (ALT), can be considered and integrated in the MO-MS- RGP model to have a more accurate and realistic estimation of parameters of the reliability growth process.
- Other optimization algorithms can be modified and utilized to solve the proposed RGP model. Also, the performance of different optimization methods, such as multiple-objective particle swarm optimization (MOPSO), in solving the proposed RGP model can be compared. In addition, the computational efficiency of the solution methodology can be analyzed in solving big-size problems.
- The proposed integrated solution methodology for the proposed RGP model, including evolutionary algorithm and data envelopment analysis, can be enhanced with the application of multiple criteria decision-making (MCDM) approaches, which compare the obtained reliability growth plans, i.e., the optimal efficient solutions, and provide the rank of solutions in terms of different criteria, such as time, cost, reliability, etc.

In the second part of this research, a new quantitative approach is proposed to model and optimize the verification and validation (V&V) activities planning process. The proposed V&V activities planning model provides an optimal set of V&V activities to be implemented considering the limited time and budget of V&V process, effectiveness of V&V activities, and sequencing of V&V activities. There are some potential extension to the proposed V&V activities planning model as listed as follows:

- Possible iteration of a V&V activity can be considered in the proposed V&V activities planning model while there is a probability of iteration for each V&V activity. In addition, the reduction in the implementation time and cost of a V&V activity after its iteration can be considered in the future models.
- Different sources of uncertainty involved in the results of the design failure modes and effect analysis (DFMEA), as input variables for the V&V activities planning model, such as: failure detectability, failure occurrence, and failure severity levels can be considered in the future works. In addition, different sources of uncertainty in the time and cost of V&V activities, as well as their effects on the detectability and occurrence of V&V activities, can be considered and modeled in the future extension of the V&V activities planning. The simulation approach can be applied to handle the sources of uncertainties in the model.
- The proposed V&V activities planning model can be modeled as a multiple objectives optimization problem in a way that in addition to maximizing the reliability improvement, the implementation time and cost of V&V process can be minimized. Multiple objectives optimization algorithms, such as NSGA-II, can be used to solve the extended multi-objective V&V activities planning problem.

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Appendix

Appendix A: List of acronyms

AMSAA	Army Material Systems Analysis Activity
BOM	Bill of Material
CAs	Corrective actions
DEA	Data Envelopment Analysis
DMU	Decision Making Unit
DoD	Department of Defense
DFMEA	Design Failure Mode and Effect Analysis
DfX	Design for X
EPA	Environmental Protection Agency
FMEA	Failure Mode Effect Analysis
NSGA-II	Fast Non-dominated Sorting Genetic Algorithm
GERT	General Evaluation and Review Technique
KCs	Key Characteristics
LNG	Liquid Natural Gas
MLE	Maximum Likelihood Estimation
MTBF	Mean Time Between Failure
MTTF	Mean Time To Failure
MOEA	Multi-objective Evolutionary Algorithm
NPD	New Product Development
NHPP	Non Homogeneous Poisson process

ORT	On-going Reliability Test
OT	Operational Testing
PERT	Project Evaluation and Review Technique
QFD	Quality Function Deployment
RGP	Reliability Growth Planning
RGT	Reliability Growth Test
RII	Reliability Improvement Index
RPN	Risk Priority Number
SCP	Set Covering Problem
V&V	Verification and Validation

Appendix B: Knee area points of Pareto optimal frontier in the MO-MS-RGP case study

DMUs	Decision variables																	objectives		
	n_{11}	n_{12}	n_{21}	n_{22}	n_{23}	n_{24}	n_{31}	n_{32}	n_{33}	t_{11}	t_{12}	t_{13}	t_{22}	t_{23}	t_{24}	t_{31}	t_{32}	t_{33}	λ_3	T	C
02	4	2	4	5	8	8	8	7	14	0.09	0.00	0.23	0.16	0.28	0.22	0.30	0.30	0.415	1.27	0.79	4558.6
03	4	2	4	5	8	8	8	7	15	0.11	0.18	0.15	0.19	0.09	0.08	0.32	0.38	0.482	1.17	0.86	4859.2
05	4	2	4	5	8	8	9	7	14	0.11	0.19	0.32	0.33	0.18	0.20	0.32	0.46	0.556	0.97	1.09	5790.8
07	4	2	4	5	8	8	8	6	14	0.07	0.06	0.27	0.15	0.34	0.22	0.29	0.28	0.401	1.21	0.82	4671.0
09	4	2	4	5	8	8	8	6	14	0.22	0.19	0.29	0.12	0.24	0.26	0.23	0.58	0.626	0.90	1.14	5973.7
12	4	2	4	5	8	8	8	7	14	0.11	0.16	0.26	0.24	0.11	0.24	0.31	0.42	0.541	1.04	0.97	5300.0
15	4	2	4	5	8	8	8	7	15	0.10	0.18	0.17	0.19	0.17	0.10	0.32	0.39	0.504	1.11	0.87	4912.7
19	4	2	4	5	8	8	8	7	14	0.10	0.01	0.27	0.16	0.22	0.19	0.29	0.33	0.417	1.24	0.79	4595.6
22	4	2	4	5	8	8	8	7	14	0.11	0.17	0.25	0.23	0.11	0.15	0.32	0.40	0.541	1.08	0.97	5285.6
24	4	2	4	5	8	8	8	7	14	0.12	0.17	0.30	0.30	0.14	0.14	0.31	0.47	0.556	1.02	1.03	5544.2
32	4	2	4	5	8	8	9	7	14	0.10	0.17	0.32	0.32	0.16	0.20	0.33	0.46	0.562	1.00	1.05	5625.9
48	4	2	4	5	8	8	8	7	14	0.09	0.00	0.23	0.16	0.27	0.22	0.30	0.31	0.418	1.26	0.79	4562.5
53	4	2	4	5	8	8	8	7	15	0.11	0.19	0.10	0.23	0.16	0.16	0.32	0.36	0.503	1.08	0.93	5145.6
60	4	2	4	5	8	8	8	7	15	0.10	0.19	0.10	0.23	0.16	0.16	0.32	0.36	0.485	1.09	0.91	5073.3
63	4	2	4	5	8	8	9	7	14	0.11	0.19	0.31	0.30	0.18	0.20	0.32	0.47	0.558	0.99	1.07	5693.2
65	4	2	4	5	8	8	8	7	14	0.11	0.19	0.30	0.30	0.14	0.14	0.32	0.47	0.557	1.02	1.05	5616.7
68	4	2	4	5	8	8	8	7	15	0.11	0.19	0.10	0.23	0.16	0.14	0.32	0.36	0.479	1.10	0.90	5023.0
71	4	2	4	5	8	8	8	7	15	0.10	0.19	0.17	0.19	0.16	0.09	0.32	0.39	0.509	1.11	0.90	5003.4
77	4	2	4	5	8	8	8	6	14	0.22	0.18	0.29	0.14	0.25	0.26	0.22	0.58	0.631	0.90	1.15	6002.3
83	4	2	4	5	8	8	9	7	14	0.11	0.19	0.32	0.31	0.19	0.19	0.32	0.46	0.556	0.98	1.07	5706.0